Volatility: Analyzing an Increasingly Prevalent Asset Class

Rajarshi Das

A project submitted in fulfillment of the requirements for the degree of MSc Risk Management and Financial Engineering and the Diploma of Imperial College London

August 25, 2011

List of Contents

Acknowledgements

I would like to thank Professor Walter Distaso for his guidance and insights on the subjects covered in this paper. I would also like to express my gratitude to my family for their continuous support over all these years.

Notation

Abbreviations

VRP	Volatility Risk Premium
LV	Long Volatility
ETF	Exchange-traded Fund
ETN	Exchange-traded Note
CBOE	Chicago Board Options Exchange

Relevant Data Series

VIX	CBOE Market Volatility Index
VX	CBOE VIX Futures
VXX	iPath S&P 500 VIX Short-Term Futures ETN
VXZ	iPath S&P 500 VIX Mid-Term Futures ETN
IEF	iShares Lehman 7-10 Year Treasury Bond ETF
GSCI	Goldman Sachs Commodity Index
VPD	CBOE VIX Premium Strategy Index

List of Tables

List of Figures

Synopsis

In this work, we scrutinize the merits of incorporating volatility as an asset class in a typical portfolio. We show that by adding volatility exposure through implied volatility, which has the benefit of increasing diversification, and volatility risk premium, which gives the portfolio return enhancements, the portfolio's risk-return profile is greatly improved. Furthermore, the combined strategies improve portfolio performance in both low-volatility regimes of market calm and provide a suitable hedge during high volatility regimes of market crises. The volatility strategies above are fairly simple to implement but we need to examine their behaviors more closely by analyzing volatility products available to investors. We show that the term structure of the VIX has played a large role in the performance of these new products since it has tended to be in contango since the inception of the tradable ETFs and ETNs based on volatility futures. The close tie between volatility and correlation is also examined while focusing on the most recent financial crisis. Through the analysis, we show that adding volatility as an asset class can provide much higher risk-adjusted returns than a portfolio consisting of the traditional assets: bonds, equities, and commodities.

1. **Introduction & Literature Review**

1.1. Volatility as an Asset Class

As financial instruments increase in complexity, investors are constantly in search of new assets with which to best optimize the risk return tradeoff. One recent advent has been the introduction of volatility as a tradable asset class. Volatility has attractive characteristics that make it a welcome addition to an investor's portfolio. It is well documented that volatility tends to be negatively correlated with equity, which immediately demonstrates the mitigation of risk by adding volatility in an equity heavy portfolio (Haugen *et al.*, 1991; Glosten *et al.*, 1993). One reason attributed to this is the "leverage effect", which posits that a market downturn lowers the equity value thus increasing leverage in the capital structure and therefore volatility (Christie, 1982; Schwert, 1989; Engle & Ng, 1993). Another theory put forward is the "volatility feedback effect" which assumes stock prices incorporate volatility and a fall in volatility lowers the future required return on equity causing stock prices to increase (Wu, 2001; Kim *et al.*, 2004). There is also the phenomenon of volatility clustering where high volatility days occur together until there is a switch to a low volatility period, when similar behavior is displayed (Mandelbrot, 1963). Volatility has also been shown to be mean-reverting, which is a very useful attribute when attempting to predict future volatility (Engle & Patton, 2001).

Recently, there has been a much wider range of products with which an investor can get exposure to volatility. First generation volatility trading was originally based on being long options via the Black-Scholes option pricing framework (Black & Scholes, 1973). However, the payoff is path-dependent and does not give exposure to purely volatility. The second generation of variance products emerged in the 1990s with variance and volatility swaps (Demeterfi *et al.*, 1999). In conjunction with these derivative products, there was also the introduction of volatility indices. Though the indices are not tradable directly, there are futures and other derivative products using the volatility index as an underlier, which have been created for investing purposes. There are now a plethora of products available that directly or indirectly trade volatility, but two main investment strategies have been highlighted: long implied volatility and long exposure to volatility risk premium (Signori *et al.*, 2009).

Long volatility is an attractive strategy for diversification purposes due to the negative correlation between volatility and equity. This allows for portfolios that are equity heavy to counteract their losses in case of a market downturn (Daigler & Rossi, 2006). The volatility risk premium, defined here as the difference between implied volatility and realized volatility, tends to have high returns for its risk (Hafner & Wallmeier, 2007). One explanation for the risk premium is the option seller, who is short volatility, has unlimited potential downside, while the option buyer, who is long volatility, has the downside capped. Hence, the option seller will demand a premium over the realized volatility and the option buyer is paying the premium to guard against market shocks (Eraker, 2008). Another reason for this premium, specifically in index options, is that the buyer pays extra because they receive protection from sudden increases in correlations among the individual constituents (Driessen *et al.*, 2009). A final explanation is that the premium is accounted for by an exogenous risk factor and if the volatility rises, the risk adjusted statistics of the portfolio itself are less impressive, and as such investors will demand a higher return (Carr & Wu, 2009).

1.2. Mean-Variance Optimization

Modern portfolio theory, pioneered by Harry Markowitz, gives a systematic approach to find an allocation which maximizes a portfolio's risk-adjusted returns. This problem can be formulated as follows based on the original theory (Markowitz, 1952):

$$\max_{w} \mu_p - \frac{\sigma_p^2}{2\lambda}$$

Where μ_p = return of portfolio and μ = expected return vector

$$\mu_p = w^{'} \mu$$

And where σ_p = standard deviation of portfolio and Ω = variance-covariance matrix of returns

$$\sigma_p = w'\Omega w$$

And λ = risk tolerance

The equation determines w = weight allocation vector

Portfolio optimization using traditional mean-variance techniques may not be enough due to the non-normality of returns data. Investor's preferences for positive skewness can be incorporated into the portfolio optimization using goal programming (Lai, 1991; Chunhachinda *et al.*, 1997). Similar extensions can be done for higher order moments.

Another common pitfall of Markowitz portfolio optimization is that weights may tend to the bounds, causing one asset to dominate the portfolio. Rational investors would not make such investment decisions and there are a few different ways to ensure that this does not occur. By adding some constraint to the weights, the optimization may give more accommodating results. The naive "1/N rule", where the weights are equally weighted, tends to only have a slightly higher estimation error and is not consistently outperformed by any of the various extensions of the mean-variance framework (DeMiguel *et al.*, 2009). There are many constraints that can be used, but it has been shown that imposing a weight constraint is actually equivalent to shrinking the variance-covariance matrix (Jagannathan & Ma, 2003). The sample variance-covariance matrix tends to have documented flaws that cause a high estimation error thus causing the mean-variance model to have misleading results. Therefore, another option is to use "shrinkage" which pulls in the variance-covariance matrix toward a central target matrix (Ledoit & Wolf, 2003) or toward the population mean (Jorion, 1985).

1.3. Volatility Trading

As mentioned above, volatility trading was first made possible with options, but this was soon expanded with the introduction of variance and volatility swaps. The range of products increased with conditional variance swaps, corridor variance swaps, and gamma swaps. The introduction of the first volatility index, VIX, in 1993 allowed for futures and derivative products to be created with the VIX as the underlier. More recently, there has been a boom in the number of ETFs and ETNs available to investors, which range from short-term, long-term, inverse, leveraged, and long-short strategies. This has made volatility trading very accessible to the regular investor and the volume of trading in many of these products have steadily surged.

Since many of these products are based on the VIX, it is important to understand how it is calculated. VIX uses near-term and next-term S&P 500 options to find the 30-day expected volatility; it is a forward looking measure. The call and put options used must have at least a week to expiry and usually have a little less than two months to maturity. A range of these options, centered on a strike price below the expected forward level of the S&P 500, are picked and weighted according to their premiums. As such, the VIX is an indicator of how much an investor is willing to pay for a put or call option on the S&P 500 at a range between the near-term and next-term dates and with strikes that are vastly higher and lower than the current market price (Chicago Board Options Exchange, 2009). In fact, for a 30-day outlook, the VIX is fairly volatile suggesting instability in the measure. The VIX has also been shown to have biases as the calculation can vary greatly even in intervals as short as 15 seconds leading to alternative measures for volatility (Andersen *et al.*, 2011).

2. Asset Analysis

2.1. Underlying Data

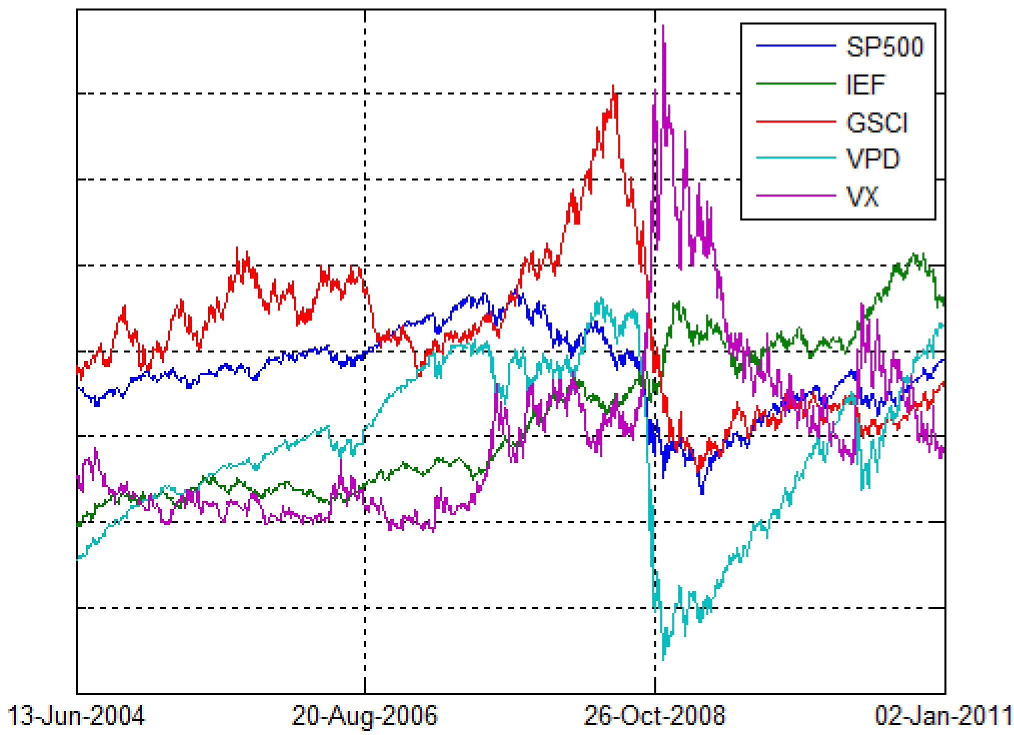
We begin by assuming an investment portfolio in the traditional asset classes: bonds, equity, and commodities. With the introduction of volatility as an asset class, the portfolio can now be enhanced. Two methods to gain exposure to volatility, long volatility (LV) and long exposure to volatility risk premium (VRP), are examined (Signori *et al.*, 2009).

For the initial analysis, the data is composed of daily data from June 15, 2004 to December 31, 2010. We use the iShares Lehman 7-10 Year Treasury Bond ETF for bonds, the S&P 500 for equities, and the S&P GSCI for commodities. The bond data measures public obligations of the US Treasury that have a maturity of 7 to 10 years and provides a reasonable time horizon for investments. Historically, commodities have been shown to be negatively correlated with stocks and bonds and have provided diversification benefits (Gorton & Rouwenhorst, 2006). However, recent evidence has shown that commodities are becoming increasingly correlated with each other and with various financial assets due to developments in securitization (Tang & Xiong, 2010). Therefore, we use the GSCI to see if it mitigates some of the diversification benefits from adding volatility exposure.

For long volatility, we use CBOE VIX futures due to the fact that the index itself is not tradable. The VIX futures data uses a continuous contract for settlement price. The CBOE VIX Premium Strategy Index is used for long exposure to the volatility risk premium. This index, VPD, consists of short one-month VIX futures which are marked-to-market daily. After one month, new VIX futures are sold. This process helps to limit risk due to a decrease in leverage compared to simply shorting VIX futures.

These innovative VIX products have been created fairly recently and the volatility products mentioned above limit our dataset to only as far back as June 15, 2004. Since our data ends in December 31, 2010, a significant portion falls within the global financial crisis and recession. As we see in Figure 1 which contains the five assets in the portfolio, this had a major impact on price levels. This inherently increases the relevance of examining the volatility, but also dictates that the results must be analyzed with this in mind.

Figure 1: Historical levels of standardized asset class proxies (Equities: SP500, Bonds: IEF, Commodities: GSCI, Implied Volatility: VX, Volatility Risk Premium: VPD), June 15, 2004 – December 31, 2010



2.2. Individual Asset Summary Statistics

In order to examine the individual assets, we determined a few key measures that would need to be calculated. We begin by using the Sharpe Ratio as the primary measure of risk-adjusted returns (Sharpe, 1966). Although Sharpe has revised the formula to use the time series of risk-free rates when calculating the excess returns, we use the original formula and make the assumption that the risk-free rate is constant over the time period for simplicity. The Sharpe Ratio is given by:

$$Sharpe\ Ratio = \frac{r_i - r_f}{\sigma}$$

Where r_i is the return of the asset, r_f is the risk-free rate, and σ is the standard deviation of the excess returns of the asset

For individual securities, the Treynor Ratio can be more useful than the Sharpe Ratio (Treynor, 1965). The Treynor ratio uses the systematic risk as measured by the asset's beta instead of total risk. Hence, it measures the return per volatility added to the portfolio as opposed to just the asset's volatility. The Treynor ratio is given by:

$$Treynor\ Ratio = \frac{r_i - r_f}{\beta_i}$$

Where r_i is the return of the asset, r_f is the risk-free rate, and β_i is the beta of the asset

In order to calculate each asset's beta, we first have to decide on what is considered the market portfolio. We are looking at asset classes and so a market portfolio is not readily apparent. Therefore, we use the tangency portfolio which consists of a mix of the five assets as a market proxy.

A summary of the individual asset class returns data can be seen in Table 1. A precursory look at the Sharpe Ratios shows that during this time period only bonds (0.518) and VRP (0.285) had acceptable risk-adjusted returns. The other three assets, LV, equities, and commodities, have negative Sharpe ratios and perform worse in that order. The Treynor Ratios have the same rankings for asset performance as the Sharpe Ratio. This is not a surprising result considering that the equity market took such a nosedive during the crisis. Though LV performs poorly, we can expect that most rational investors would probably not remain long volatility after VIX had reached its peak levels. The annualized geometric means of the returns confirm the attractiveness of the assets as before. The standard deviations for all assets, except bonds, are quite large due to the market shocks from the financial crisis. Both volatility strategies have relatively large maximum daily gains and losses, which is explained by their hypersensitivity to poor market conditions. Interestingly, equities, bonds, and LV all have a positive skew. The large positive skew for LV suggests that it hedges the asymmetry that one would expect from a crash. Not surprisingly, the kurtosis of every asset is quite high suggesting very fat tails. The higher order moments, in addition to the results of the Jarque-Bera Normality

Test at a 5% significance level demonstrate the data is clearly not normal (Jarque & Bera, 1987).

Table 1: Summary statistics of daily returns data for the five assets, June 15, 2004 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
Daily Geometric Mean	0.0064%	0.0231%	-0.0017%	0.0332%	0.0081%
Annual Geometric Mean	1.62%	6.00%	-0.43%	8.74%	2.07%
Max Daily Gain	11.58%	3.43%	7.48%	17.57%	29.46%
Max Daily Loss	-9.04%	-1.78%	-8.29%	-18.15%	-18.07%
Annual Standard Deviation	22.36%	7.21%	27.67%	22.78%	56.58%
Skewness	0.014	0.222	-0.109	-0.895	1.204
Kurtosis	14.017	5.956	4.940	41.860	9.100
Success Rate	54.97%	51.76%	51.03%	57.82%	44.30%
Sharpe Ratio	-0.028	0.518	-0.097	0.285	-0.003
Beta	0.285	0.586	0.395	1.336	2.351
Treynor Ratio	-2.229	6.378	-6.788	4.849	-0.080
Jarque-Bera Normality Test (1 = Not Normal)	1	1	1	1	1

2.3. Correlations and Dependencies

The correlation matrix in Table 2 demonstrates succinctly the diversification benefits of these assets. Between the traditional assets, bonds are already negatively correlated with both equities (-0.387) and commodities (-0.171). The diversification benefits of LV become immediately apparent as it is very negatively correlated with equities (-0.657) and commodities (-0.194) which suggest it is a better hedge than just bonds. LV and bonds have a slight positive correlation (0.274). The VRP strategy is negatively correlated with bonds (-0.280) and also very negatively correlated with the LV strategy (-0.661) suggesting the two volatility strategies are good hedges against each other.

Table 2: Correlation matrix of daily returns, June 15, 2004 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
Equity		-0.387	0.309	0.777	-0.657
Bond			-0.171	-0.280	0.274
Commodity				0.324	-0.194
VRP					-0.661
LV					

We can expect these highlighted characteristics to play a strong role in the portfolio optimization. The LV strategy has the distinct advantage of having high returns when the equity market is going down. Due to recent increases in correlation between commodities and equity, the diversification benefit of the LV strategy is even greater. On the other hand, the VRP strategy has an augmented risk and return compared to equities and commodities. As such, it can enhance the returns of the portfolio while also receiving the diversification benefits of the LV strategy. This is very apparent in times of crises as the two volatility strategies tend to peak in opposite directions. The polarized behavior is quite beneficial to prevent large downside drops in portfolio value.

3. Portfolio Construction & Mean-Variance Optimization

3.1. Original Dataset

Using basic Markowitz mean-variance portfolio optimization, we construct four different portfolios to explore the effects on the efficient frontier. These portfolios include:

Portfolio 1 (P1): Bond, Equity, and Commodity

Portfolio 2 (P2): Bond, Equity, Commodity, and LV

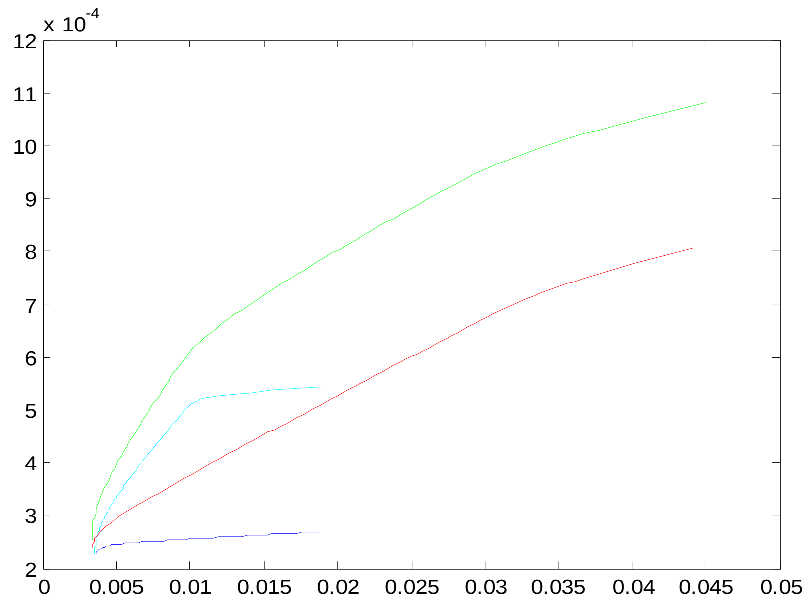
Portfolio 3 (P3): Bond, Equity, Commodity, and VRP

Portfolio 4 (P4): Bond, Equity, Commodity, LV, and VRP

The optimization constrains the weights from -1 to 1, hence allowing short selling. In the initial analysis, we examine in-sample evidence and do not include transaction costs or any rebalancing.

The benefits of the volatility strategies are immediately apparent by the movements in the efficient frontier in Figure 2. Adding the individual volatility strategies show solid improvements in the risk and return profile. P2, the LV strategy (red), shows an increase in returns while lowering the overall risk. P3, the VRP strategy (cyan) greatly enhances the returns. Both of these were predicted by the summary statistics above. Finally, adding both strategies (green) vastly improves the efficient frontier compared with the original portfolio of equities, bonds, and commodities.

Figure 2: Mean-variance optimization efficient frontiers of the 4 different portfolios, June 24, 2004 – December 31, 2010



In Table 3, we can see the numerical values behind the risks and returns of both the risky and optimal portfolios for each of the asset groupings. The return enhancement of the VRP strategy makes the return to risk ratio of the portfolio (0.0545) more attractive than the diversification benefit of the LV strategy (0.0510), though both strategies together perform even more admirably (0.0670) compared to the original portfolio with no volatility assets (0.0458).

Table 3: Portfolio risks and returns for optimal risky and optimal overall portfolio, June 24, 2004 – December 31, 2010

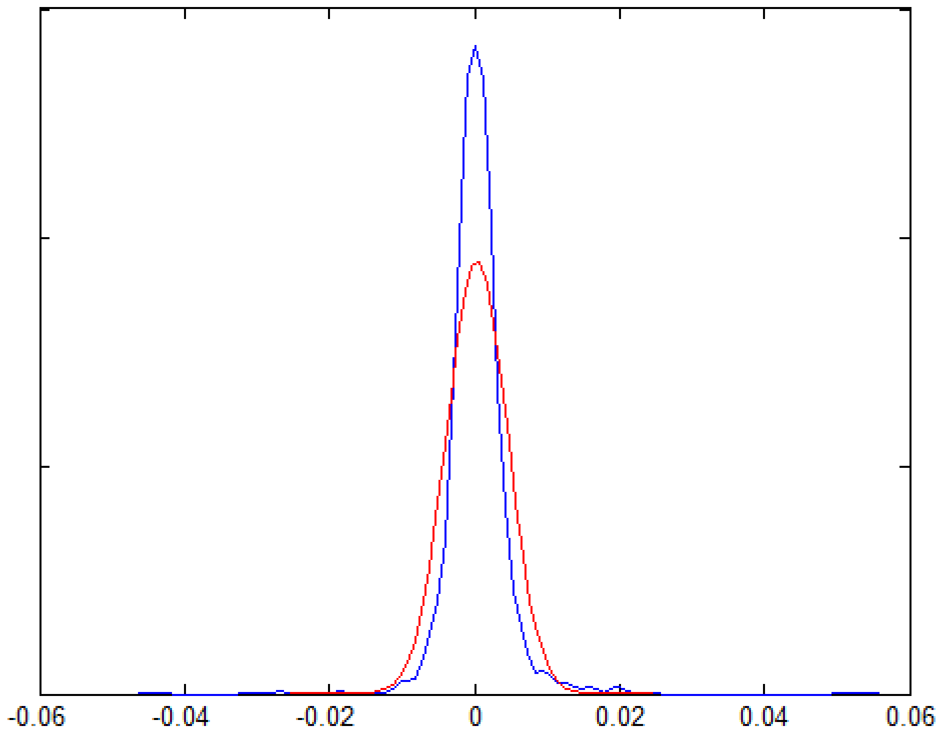
	P1	P2	P3	P4
Risky Risk	0.0036	0.0045	0.0035	0.0042
Risky Return	0.0002	0.0003	0.0003	0.0004
Risky Fraction	3.65	3.71	4.44	4.95
Overall Risk	0.0131	0.0165	0.0157	0.0209
Overall Return	0.0006	0.0009	0.0008	0.0014
Overall Return / Overall Risk	0.0458	0.0545	0.0510	0.0670

Table 4 provides summary statistics on the performance of the portfolios. The Sharpe Ratio of the full portfolio with both volatility strategies is the highest (1.105), followed by the VRP enhanced portfolio (0.790), LV enhanced portfolio (0.752), and the basic portfolio (0.618). The volatility reduction of the portfolios is apparent by the lower, and thus more reasonable, values for maximum daily gain and loss. The positive skewness of the portfolio (0.6) with all assets is greatly beneficial to the average investor who prefers a positive fat tail. The kurtosis (32.15), on the other hand, is extremely high for the portfolio, which leads to some other questions. To explore it further, Figure 3 shows the nonparametric density (blue) alongside the normal distribution (red) with the same mean and variance, for the full portfolio. The high kurtosis can be attributed to the extreme peak of the density, along with the bumps that are visible in the tails. This kurtosis risk can be directly attributed to the financial crisis and is an integral part of the behavior of the volatility strategies.

Table 4: Summary statistics of daily returns of each portfolio, June 24, 2004 – December 31, 2010

	P1	P2	P3	P4
Daily Geometric Mean	0.0223%	0.0300%	0.0249%	0.0344%
Annual Geometric Mean	5.77%	7.86%	6.48%	9.04%
Max Daily Gain	3.15%	4.95%	2.73%	5.41%
Max Daily Loss	-1.89%	-4.27%	-1.98%	-4.45%
Annual Standard Deviation	5.69%	7.08%	5.62%	6.69%
Skewness	0.228	-0.228	0.617	0.600
Kurtosis	8.515	22.013	7.870	32.150
Success Rate	53.94%	55.03%	51.27%	52.55%
Sharpe Ratio	0.618	0.790	0.752	1.015

Figure 3: Return distributions for full portfolio (P4) – Normal distribution shown in red and non-parametric distribution shown in blue, June 24, 2004 – December 31, 2010



The allocations in each asset for the portfolios are shown in Table 5. The initial portfolio has 13.3% in equities, 84.8% in bonds, and 1.9% in commodities. Inclusion of the VRP strategy has the most impact on equities (-10.6%) and commodities (-0.6%) while there is 30.6% in VRP and 80.5% in bonds. Since VRP has a high positive correlation with equities and moderate positive correlation with commodity, the higher risk-adjusted return makes equities and commodities redundant. The portfolio including LV has 23% in equities, 68.3% in bonds, 1.4% in commodities, and 7.3% in LV. Since LV was slightly correlated with bonds, the weight shifted away from bonds. The full portfolio has low weights in equities (1.1%) and commodities (-1.4%) and a more balanced 57.4% in bonds, 32.8% in VRP, and 10.1% in LV. The extreme weights calculated by the mean-variance optimization are questionable; it is unlikely that rational investors would ignore equities in their portfolio. Possible solutions include shrinking the variance-covariance matrix or adding some constraints to the weights.

Table 5: Weight allocations of each asset in the portfolios, June 24, 2004 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
P1	13.3%	84.8%	1.9%		
P2	-10.6%	80.5%	-0.6%	30.6%	
P3	23.0%	68.3%	1.4%		7.3%
P4	1.1%	57.4%	-1.4%	32.8%	10.1%

3.2. High & Low Volatility Regimes

Since it is clear from the above analysis that the portfolio with all five assets far outperforms the portfolios with a subset of the assets, for the remainder of the paper we will focus only on this full portfolio of all assets. We now divide the time period in two, into low volatility (pre-crisis) and high volatility (post-crisis) periods. The date used to split the data is October 10, 2008 which is picked based on a 50-day simple moving average that marks when the VIX jumps above a cutoff of 30. This cutoff has been chosen as being approximately 50% greater than the average value for VIX since inception (about 21.06) to mark a very large jump into a high volatility regime. We use mean-variance optimization with the assumptions as above (constraints -1 to 1, no transaction costs), including all assets, on the low volatility period and high volatility period separately. The full period is also included for comparison.

In Table 6, the summary statistics are shown for the 3 portfolios. Immediately, the most interesting feature is that the high volatility portfolio performs the best of the three strategies with the highest Sharpe Ratio (1.509) and success rate (55.26%). The high volatility regime also boasts the lowest maximum daily loss (-2.99%), the highest positive skew (0.923), and lowest kurtosis (10.390). This is a surprisingly optimistic result since these are the opposite characteristics of a high volatility regime, particularly since this was during the financial crisis. Comparatively, in the low volatility regime, the Sharpe ratio is lower (0.912), the portfolio has a negative skew (-0.495), and it has higher kurtosis (19.840), all of which are less desirable.

Table 6: Summary statistics of daily returns for pre-crisis period, June 24, 2004 – October 10, 2008, and post-crisis period, October 11, 2008 – December 31, 2010

	Low volatility	High volatility	All
Daily Geometric Mean	0.0378%	0.0578%	0.0344%
Annual Geometric Mean	10.00%	15.68%	9.04%
Max Daily Gain	2.81%	4.35%	5.41%
Max Daily Loss	-4.34%	-2.99%	-4.45%
Annual Standard Deviation	7.25%	10.34%	6.69%
Skewness	-0.495	0.923	0.600
Kurtosis	19.840	10.390	32.150
Success Rate	53.67%	55.26%	52.55%
Sharpe Ratio	0.912	1.509	1.015

The weight allocations of the assets in the low and high volatility regimes are displayed in Table 7. In the pre-crisis period, the weights are as follows: equities (-25.9%), bonds (41%), commodities (3.2%), VRP (68.4%), and LV (13.3%). Table 8 contains summary statistics of the five assets in the low volatility period, while Table 9 contains the correlation matrix. Once again, it is the risk enhancement of the VRP strategy that dominates the portfolio. This makes commodities virtually useless and makes equities heavily shorted likely due to their positive correlations with VRP. Interestingly, the VRP has a low Sharpe Ratio (-0.095) yet the highest Treynor Ratio (20.614). The negative beta for the VRP strategy suggests that it is actually reducing risk of the portfolio and explains the apparent contradiction between the two ratios. VRP is also positively correlated with equities (0.785) and negatively correlated with bonds (-0.252) and LV (-0.710) giving it merit as a hedge. This is an unexpected result because in general, the VRP strategy is similar to selling insurance (makes small positive returns during low volatility and large losses in high volatility) yet its behavior here is the opposite. Both bonds (0.348) and LV (0.323) have high Sharpe Ratios explaining the positive allocations in each.

Table 7: Weight allocations of each asset in the portfolio for pre-crisis period, June 24, 2004 – October 10, 2008, and post-crisis period, October 11, 2008 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
Low Volatility	-25.9%	41.0%	3.2%	68.4%	13.3%
High Volatility	25.4%	65.7%	-19.9%	21.4%	7.4%
All	1.1%	57.4%	-1.4%	32.8%	10.1%

Table 8: Summary statistics of daily returns data for the five assets during pre-crisis period, June 24, 2004 – October 10, 2008

	Equity	Bond	Commodity	VRP	LV
Daily Geometric Mean	-0.0211%	0.0213%	0.0157%	0.0083%	0.0733%
Annual Geometric Mean	-5.18%	5.52%	4.03%	2.11%	20.29%
Max Daily Gain	5.42%	1.92%	6.79%	6.15%	29.46%
Max Daily Loss	-8.79%	-1.78%	-8.29%	-8.41%	-14.26%
Annual Standard Deviation	16.56%	6.14%	24.93%	13.53%	52.35%
Skewness	-1.158	-0.025	-0.179	-1.495	1.702
Kurtosis	13.244	4.912	4.672	22.149	13.270
Success Rate	54.59%	51.38%	51.38%	57.25%	45.50%
Sharpe Ratio	-0.518	0.348	0.026	-0.095	0.323
Sortino Ratio	-0.036	0.033	0.013	-0.002	0.059
Beta	-0.584	0.430	0.719	-0.062	5.471
Treynor Ratio	14.689	4.963	0.893	20.614	3.089

Table 9: Correlation matrix of daily returns during pre-crisis period, June 24, 2004 – October 10, 2008

	Equity	Bond	Commodity	VRP	LV
Equity		-0.389	0.041	0.785	-0.666
Bond			-0.018	-0.252	0.279
Commodity				0.097	-0.020
VRP					-0.710
LV					

It is important to note that the cutoff was picked based on when the VIX index reached a certain value (chosen as 30). Hence, the LV strategy (where VIX is the underlier), and possibly the VRP strategy, will surely have radically different behaviors on either side of this divide. On the other hand, the financial crisis started having effects earlier than the cutoff of October 10, 2008. Although, it is widely accepted that in October 2008 markets degenerated extremely quickly, the various causes that triggered the crisis started at different times (Taylor, 2008). The major impacts on the credit and equity markets began in the middle of 2007. In fact the S&P 500 and commodities both started trending downwards earlier, giving some explanation to their allocations in the portfolio. We examine a different cutoff later in the analysis and get an idea of how sensitive portfolio optimization can be to even a slight change in the data.

The post-crisis “recovery” allocations are different: equities (25.4%), bonds (65.7%), commodities (-19.9%), VRP (21.4%), and LV (7.4%). Table 10 and Table 11 contain the statistics of each asset and the correlation matrix in the high volatility period, respectively. Between the high and low volatility period the only correlations that changed were that of commodities with the other assets. However, the overall performance of the assets changed dramatically. The Sharpe Ratios are much higher now for equities (0.508), bonds (0.696), and VRP (0.562) and much lower for commodities (-0.370) and LV (-0.396), which slightly polarizes the allocation. This allocation is also interesting due to the fact that the VRP strategy is rising steadily while LV is falling steadily as markets slowly crawl back up, yet LV has a positive weight. Examining the Treynor Ratios draws the same conclusions. Equities (7.867), Bonds (9.354), and VRP (7.900) have high Treynor Ratios and they receive most of the asset allocation. The LV strategy has a negative beta yet a positive Treynor Ratio, and as such reduces portfolio risk via diversification. Perhaps this explains the positive allocation in LV despite its apparent negative returns. The commodities asset class has a negative Treynor Ratio and a positive beta, revealing that it had a very poor performance; hence the negative weight allocation.

Table 10: Summary statistics of daily returns data for the five assets during post-crisis period, October 11, 2008 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
Daily Geometric Mean	0.0577%	0.0244%	-0.0509%	0.0706%	-0.1155%
Annual Geometric Mean	15.65%	6.34%	-12.04%	19.46%	-25.27%
Max Daily Gain	11.58%	3.43%	7.48%	17.57%	14.34%
Max Daily Loss	-9.04%	-1.65%	-8.29%	-18.15%	-18.07%
Annual Standard Deviation	30.63%	8.98%	32.81%	34.46%	63.97%
Skewness	0.307	0.338	-0.117	-0.682	0.664
Kurtosis	9.467	5.263	4.794	22.702	4.790
Success Rate	55.62%	52.41%	50.27%	58.82%	42.07%
Sharpe Ratio	0.508	0.696	-0.370	0.562	-0.396
Sortino Ratio	0.058	0.068	-0.020	0.059	-0.014
Beta	1.978	0.668	0.839	2.453	-0.937
Treynor Ratio	7.867	9.354	-14.457	7.900	27.055

Table 11: Correlation matrix of daily returns during pre-crisis period, June 24, 2004 – October 11, 2008

	Equity	Bond	Commodity	VRP	LV
Equity		-0.387	0.534	0.782	-0.676
Bond			-0.309	-0.296	0.269
Commodity				0.497	-0.405
VRP					-0.696
LV					

In the market conditions of a high volatility regime, it is important to make a distinction between upside and downside risk. The LV and VRP strategies, especially, peak at the beginning of the time period and then slowly trend back towards some central value. The Markowitz framework uses symmetric risk, due to its reliance on just the first two moments. Similarly, the Sharpe Ratio also, counter-intuitively, treats upside and downside volatility equally. The mean-variance optimization has been extended to penalize downside risk more, but most of these alternatives have been rejected in favor of the original framework (King, 1993). Therefore, instead of using an alternative to the Markowitz framework, we aim for a simpler objective: calculate a different portfolio performance indicator that incorporates the asymmetry of risk. We use the Sortino Ratio which also measures the risk-adjusted return but only penalizes downside risk that falls below a certain minimum accepted return (Sortino & Price, 1994). The Sortino Ratio is given by:

$$Sortino\ Ratio = \frac{r - MAR}{DR}$$

Where r is the return of the asset, MAR is the minimum acceptable return (we use the risk-free rate), and DR is downside risk.

$$DR = \left(\int_{-\infty}^{MAR} (MAR - x)^2 f(x) dx \right)^2$$

And where $f(x)$ is the probability density function of the returns

However, in the high volatility case, the Sortino Ratio ranking remains the same for the assets and doesn't shed any additional light on the discrepancy between the negative Sortino ratio and the positive allocation of the LV strategy, other than LV's role in risk reduction as discussed above. The correlation matrix shows that LV is negatively correlated with Equity, Commodities, and VRP so it still maintains its diversification benefit, again explaining the positive weight in the strategy.

The exogenously chosen cutoff of 30, which split the data at October 10, 2008, did not seem to account for a large portion of the financial crisis. Instead, now we choose a divide in the data using more of a qualitative approach. VIX demonstrates some of the basic characteristics of volatility highlighted above: mean reverting and volatility clustering. Examining the VIX index, we can see that around August 2007 the volatility begins increasing leaving behind the mean level which is sub-20 during times of market stability and growth. From here on, the volatility remains high while demonstrating spikes intermittently. Based on this observation, if we use a cutoff of 20, the high volatility regime begins at August 27, 2007. This date also represents one of the first large dips in the S&P 500 after about 4 years of strong performance. However, it is still a few months until the market takes a complete nosedive.

This aligns with many of the happenings in the market during that time period. Although the evidence of a crisis was apparent to those in the financial industry much earlier, there was a degree of information asymmetry that kept the public unaware for the most part. We look at a general timeline of the crisis to grasp better the forces dictating market movements (Federal Reserve Bank of St. Louis, n.d.). The first sign was the bursting of the housing bubble which led to a sharp drop in existing home prices throughout the United States. This was followed by a collapse in the mortgage industry in the early summer of 2007. This began a long series of disappointing announcements by large institutions and corporations ranging from heavy losses to filing for bankruptcy that would prevail throughout the crisis. By late summer of 2007, everyone was more or less aware of the impending crash. This also coincided with the peak of the credit boom and the consequent drop in lending reflected both scale back measures in firms and tightening of liquidity (Ivashina & Scharfstein, 2010). The dividing date chosen above, August 27, 2007, more or less marks the beginning of a scramble to save the economy. Using this new cutoff date, we can perhaps gain different insights into how volatility reacted to the happenings during this market turmoil.

Table 12 provides summary statistics for the low and high volatility periods and Table 13 gives the allocations in each asset class. We compare these results to the previous cutoff and find that the low volatility period, with a Sharpe Ratio of 2.143, now outperforms the high volatility period (0.992). This is expected because we chose the cutoff so that now the bulk of the crisis falls into the latter period. The weight allocations in the low volatility period are no longer heavy on shorting equity, but still have a heavy weight in VRP (69.2%), followed by bonds (20.8%), and LV (10%). We have a continuing issue with the unreasonable weights in certain assets; this is just not realistic for long term investors. The high volatility period is a lot heavier on bonds (69.3%) followed by VRP (28.3%) and LV (8.3%).

Table 12: Summary statistics of daily returns for pre-crisis period, June 24, 2004 – August 27, 2007, and post-crisis period, August 28, 2007 – December 31, 2010

	Low volatility	High volatility	All
Daily Geometric Mean	0.0492%	0.0340%	0.0344%
Annual Geometric Mean	13.19%	8.96%	9.04%
Max Daily Gain	2.78%	4.48%	5.41%
Max Daily Loss	-1.81%	-3.39%	-4.45%
Annual Standard Deviation	4.42%	8.13%	6.69%
Skewness	2.230	0.281	0.600
Kurtosis	25.180	13.443	32.150
Success Rate	58.06%	53.14%	52.55%
Sharpe Ratio	2.143	0.992	1.015

Table 13: Weight allocations of each asset in the portfolios for pre-crisis period, June 24, 2004 – August 27, 2007, and post-crisis period, August 28, 2007 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
Low Volatility	-0.6%	20.8%	0.7%	69.2%	10.0%
High Volatility	-3.2%	69.3%	-2.7%	28.3%	8.3%
All	1.1%	57.4%	-1.4%	32.8%	10.1%

Similar summaries as above for the low volatility data can be seen in Table 14 and Table 15. As per the Treynor Ratio and the Sharpe Ratio, VRP performs the best with equities and bonds following behind. It is also interesting to note that in this period, the correlation between commodities and equities is actually negative (and the correlation between commodities and bonds positive). This has been shown in previous literature but is thought to no longer hold as strongly. The statistics for high volatility data can be found in Table 16 and Table 17. Bonds and VRP have positive Sharpe Ratios, while the remaining asset classes perform poorly judging by their risk adjusted returns. The surprisingly high Treynor Ratios for equities and commodities along with their negative betas, point out that the assets performed extremely poorly in the high volatility regime. Since we know the cause of this high Treynor Ratio and know that these assets are generally not used as insurance for market portfolios (which would also have negative betas), it is safe to ignore this value and not base any rankings on it. The correlations within all asset classes have changed significantly in the crisis period. As before, we see evidence of higher absolute correlations as all assets begin to move together as the market tumbles. Both volatility strategies still maintain their unique characteristics (VRP with return enhancement and LV with diversification) that make them a beneficial addition to the portfolio regardless of market behavior.

While the conclusion we draw on the behavior of volatility is somewhat similar for both cutoffs, we also see some unique characteristics caused by the high sensitivity of this type of analysis from the change in cutoff date. For an investor who was actively managing a portfolio through the crisis, the choice of the divide is crucial. This demonstrates that though we can easily analyze the portfolio in hindsight, market timing would have made a large difference in the portfolio allocation and subsequent risk management at the time.

Table 14: Summary statistics of daily returns data for the five assets during pre-crisis period, June 24, 2004 – August 27, 2007

	Equity	Bond	Commodity	VRP	LV
Daily Geometric Mean	0.0321%	0.0179%	0.0196%	0.0528%	0.0373%
Annual Geometric Mean	8.44%	4.60%	5.08%	14.22%	9.87%
Max Daily Gain	2.46%	1.37%	6.79%	6.15%	29.46%
Max Daily Loss	-3.47%	-1.00%	-4.64%	-3.64%	-14.26%
Annual Standard Deviation	11.06%	4.84%	22.84%	8.74%	48.51%
Skewness	-0.330	0.053	0.116	0.476	2.276
Kurtosis	4.768	3.621	3.433	27.704	19.178
Success Rate	56.08%	50.87%	51.24%	58.93%	44.79%
Sharpe Ratio	0.428	0.185	0.060	1.202	0.127
Sortino Ratio	0.040	0.018	0.016	0.104	0.040
Beta	0.607	0.100	0.401	1.028	1.746
Treynor Ratio	7.797	8.956	3.414	10.228	3.527

Table 15: Correlation matrix of daily returns during pre-crisis period, June 24, 2004 – August 27, 2007

Table 15	Equity	Bond	Commodity	VRP	LV
Equity		-0.121	-0.021	0.691	-0.592
Bond			0.049	-0.094	0.101
Commodity				0.058	-0.022
VRP					-0.705
LV					

Table 16: Summary statistics of daily returns data for the five assets during post-crisis period, August 28, 2007 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
Daily Geometric Mean	-0.0192%	0.0284%	-0.0217%	0.0134%	-0.0143%
Annual Geometric Mean	-4.73%	7.42%	-5.33%	3.43%	-3.54%
Max Daily Gain	11.58%	3.43%	7.48%	17.57%	17.84%
Max Daily Loss	-9.04%	-1.78%	-8.29%	-18.15%	-18.07%
Annual Standard Deviation	29.33%	8.91%	31.59%	30.68%	63.38%
Skewness	0.055	0.200	-0.174	-0.711	0.703
Kurtosis	9.166	4.699	4.781	24.683	5.017
Success Rate	53.85%	52.66%	50.89%	56.69%	43.91%
Sharpe Ratio	-0.192	0.733	-0.197	0.083	-0.070
Sortino Ratio	-0.004	0.070	-0.004	0.020	0.024
Beta	-0.192	0.916	-0.035	0.995	2.090
Treynor Ratio	29.330	7.129	178.964	2.548	-2.120

Table 17: Correlation matrix of daily returns during post-crisis period, August 28, 2007 – December 31, 2010

	Equity	Bond	Commodity	VRP	LV
Equity		-0.443	0.409	0.789	-0.712
Bond			-0.256	-0.315	0.348
Commodity				0.400	-0.284
VRP					-0.710
LV					

3.3. Rebalancing Portfolio

We now use the same mean-variance framework on a rebalancing portfolio over the same dataset (June 15, 2004 to December 31, 2010). The weights will be calculated on a 30 day piecewise basis and used on the next out of sample period. This will provide a much more realistic and robust look at the merits of the volatility strategies, since we no longer use the sample data itself for the back-testing.

The rebalancing is done over 30 days, which is actually over a month as only business days are counted in the sample. This number has been picked as a starting point and can be changed if necessary. The piecewise window refers to the following: the first 30 days will provide data for the portfolio optimization. The weights calculated as such will then be used to allocate the wealth in the portfolio starting from the 31st to the 60th day. During this period, new weights are also calculated for next period's wealth allocation. Finally, we also include a blanket transaction cost of 0.1% on the wealth of the portfolio. This is meant to cover both commissions and slippage in a simple manner without adding extra intricacies at this stage. Transaction costs are deducted every 30 days when the portfolio is rebalanced.

Though the mean-variance framework remains, seven different portfolio constraints are examined in process. These include long only (constraints of 0% to 100%), 120/20 (constraints of -20% to 120%), 130/30, 140/40, 150/50, pure long/short (constraints of -100% to 100%), and using Ledoit & Wolf's shrinkage on the variance-covariance matrix. The shrinkage estimator of the covariance is calculated as shown:

$$\Sigma_{shrinkage} = \delta F + (1 - \delta) S$$

Where δ is the shrinkage constant, F is the shrinkage target, and S is the sample variance-covariance matrix

The interested reader can find the calculation for the optimal shrinkage constant in the original paper (Ledoit & Wolf, 2003).

The above constraints are per asset and not on the actual portfolio as a whole, which allows more freedom for the optimizer. As such, the entire portfolio may not be long and short in the same ratio as the constraints.

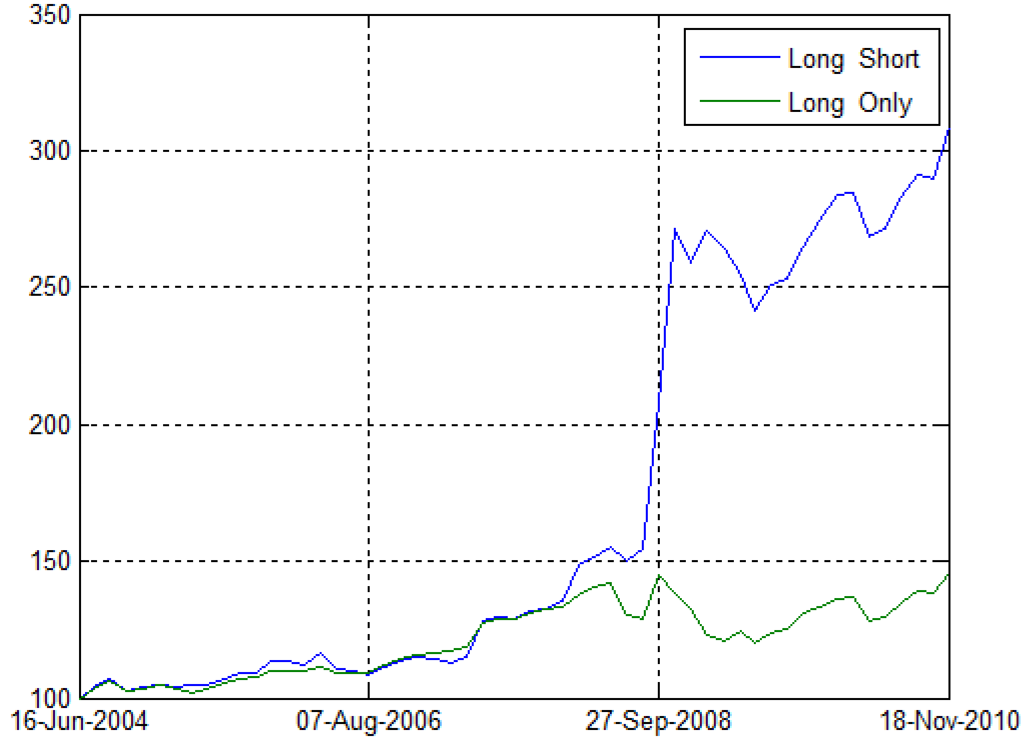
Table 18 consists of the relevant statistics for the various portfolios. Since these tests are out of sample, the final wealth of an investor becomes more relevant. This is calculated assuming an initial investment of 100. The benefits of shorting are immediately clear by this final wealth calculation since all the constraints which included shorting, perform significantly better than those without shorting. In fact, looking just at the wealth, the pure long/short performs extremely well during the given time period. Figure 4 displays the wealth path of both the long only and the pure long/short portfolios for comparison. There is an apparent jump in the wealth around August 2008 in the shorting portfolio.

Looking at the weight allocations of the portfolio, it is clear that the success was mainly due to polarized weights (fully shorting VRP and commodities) and consequently facing a major downturn in those assets in the next period. Shorting provides this benefit by allowing an extra avenue to generate returns. There is also the element of risk to consider since this could just be luck. Judging by the Sharpe Ratio, the 140/40 strategy performs the best (1.172). On the other hand, the Sortino Ratio is highest for the 150/50 strategy (0.108). Both strategies highlight the benefits of shorting in a volatile market without increasing their risk profile too severely. These strategies also have a positive skew and a lower kurtosis than the basic, long-only portfolio. The shrinkage method seems to have a similar effect as the long only strategy though with even poorer results. One reason may be that shrinkage lessens the extremes generated through normal mean-variance optimization. The portfolio will not have polarized weight allocations, but also cannot make large gains (or losses) as the shorting portfolios do. The shrinkage parameter is also extremely low, suggesting the shrinkage matrix didn't actually change much from the sample variance-covariance matrix.

Table 18: Summary statistics of daily returns data for various constraints in a rebalancing portfolio, June 15, 2004 – December 31, 2010

	Long only	Full Shorting	120/20	130/30
Daily Geometric Mean	0.0279%	0.0797%	0.0479%	0.0545%
Annual Geometric Mean	7.28%	22.24%	12.82%	14.70%
Max Daily Gain	5.33%	15.15%	4.32%	4.67%
Max Daily Loss	-6.36%	-13.92%	-4.28%	-4.38%
Annual Standard Deviation	11.11%	18.58%	10.02%	11.02%
Skewness	-0.260	1.102	0.029	0.169
Kurtosis	18.362	45.098	10.035	11.102
Success Rate	56.85%	54.94%	55.19%	55.49%
Sharpe Ratio	0.452	1.076	1.055	1.130
Sortino Ratio	0.043	0.104	0.096	0.104
Final Wealth	145.880	308.387	198.280	217.703
	140/40	**150/50**	**Shrinkage**	
Daily Geometric Mean	0.0605%	0.0660%	0.0270%	
Annual Geometric Mean	16.47%	18.09%	7.05%	
Max Daily Gain	6.02%	7.51%	6.02%	
Max Daily Loss	-5.02%	-6.50%	-9.56%	
Annual Standard Deviation	12.13%	13.52%	12.97%	
Skewness	0.285	0.375	-0.643	
Kurtosis	12.981	15.446	23.677	
Success Rate	55.31%	55.49%	56.42%	
Sharpe Ratio	1.172	1.171	0.369	
Sortino Ratio	0.108	0.108	0.037	
Final Wealth	237.407	256.652	144.221	

Figure 4: Wealth path of long/short and long only portfolios, June 15, 2004 – December 31, 2010



3.4. Extended Dataset

One shortcoming of the analysis above is the limit of the size of the dataset used. To examine the unique effects of volatility further, we now extend the dataset to a longer timeframe. We focus on just pure volatility which is traded via VIX futures. Since the inception date of the VIX futures is March 26, 2004, we extend the data based on its relation to the VIX underlier. Table 19 shows linear regression results of the VIX futures on the VIX index. We see that the futures track the VIX very closely with an adjusted R square of 0.926. Using the beta coefficients, the VIX future time series can be extended all the way back to January 2, 1990.

Table 19: Regression results for VIX futures estimation, June 15, 2004 – December 31, 2010

	Alpha	Beta	Adjusted R Square
Coefficient	4.400	0.825	0.926
t statistic	34.361	150.813	

A summary of statistics for the individual assets from January 2, 2004 to December 31, 2010 are seen in Table 20. The results are slightly different than we had in the shortened dataset. The main takeaway is the rankings of the asset's risk adjusted returns based on their Sharpe Ratio and Treynor Ratio from highest to lowest: bonds, equities, commodities, and LV. The latest financial crisis clearly doesn't have as much of an effect on equities and commodities, both of which now have positive risk adjust returns. The correlation of the asset classes are in Table 21. These values are also fairly close to that of the original dataset, although most correlations seem to be less extreme (closer to zero). This can also be attributed to the financial crisis, where it can be expected that asset classes will increase in correlation towards one (or decrease towards negative one) as the markets move drastically in one direction.

Table 20: Summary statistics of daily returns data for the four assets, January 2, 2004 – December 31, 2010

	Equity	Bond	Commodity	LV
Daily Geometric Mean	0.0237%	0.0303%	0.0177%	0.0004%
Annual Geometric Mean	6.14%	7.92%	4.56%	0.11%
Max Daily Gain	11.58%	6.83%	7.90%	43.44%
Max Daily Loss	-9.04%	-5.80%	-16.83%	-26.16%
Annual Standard Deviation	18.58%	9.91%	22.12%	76.49%
Skewness	-0.008	0.074	-0.407	1.040
Kurtosis	12.007	8.653	10.103	9.176
Success Rate	53.07%	43.66%	51.01%	47.40%
Sharpe Ratio	0.135	0.432	0.042	-0.046
Beta	0.645	0.711	0.524	3.917
Treynor Ratio	3.892	6.027	1.771	-0.901
Jarque-Bera Normality Test (1 = Not Normal)	1	1	1	1

Table 21: Correlation matrix of daily returns, January 2, 2004 – December 31, 2010

	Equity	Bond	Commodity	LV
Equity		-0.122	0.105	-0.713
Bond			-0.131	0.071
Commodity				-0.066
LV				

Using the mean-variance framework, we are now able to assess the merits of the portfolio with bonds, equities, commodities, and volatility. The results of the portfolio optimization with the original dataset are compared to that of the extended dataset in Table 22. Overall, the portfolio performs better in the long run with a higher Sharpe Ratio (0.9282). The skewness, though positive as investor's would prefer, is comparatively less. The extended portfolio also has a lower kurtosis. Most importantly, these results show that the benefits of including volatility hold in the 20-year investment horizon as well. The weights in each asset are shown in Table 23. The weights are now more balanced with a higher allocation in equities (41.3%), commodities (6.5%), and LV (10.2%) and a lower allocation in bonds (41.9%). All in all, the longer dataset still maintains similar results to before, giving additional credence to adding volatility as an asset class. However, there is a degree of moderation in many of the values that falls more in line with other empirical studies of long term investment decisions.

Table 22: Summary statistics of daily returns data of extended dataset, January 2, 2004 – December 31, 2010

	Extended Data	Original Data
Daily Geometric Mean	0.0385%	0.0249%
Annual Geometric Mean	10.20%	6.48%
Max Daily Gain	3.48%	2.73%
Max Daily Loss	-2.76%	-1.98%
Annual Standard Deviation	7.07%	5.62%
Skewness	0.460	0.617
Kurtosis	6.792	7.870
Success Rate	52.20%	51.27%
Sharpe Ratio	0.928	0.752

Table 23: Weight allocations of each asset in extended dataset, January 2, 2004 – December 31, 2010

	Equity	Bond	Commodity	LV
Extended Data	41.3%	41.9%	6.5%	10.2%
Original Data	23.0%	68.3%	1.4%	7.3%

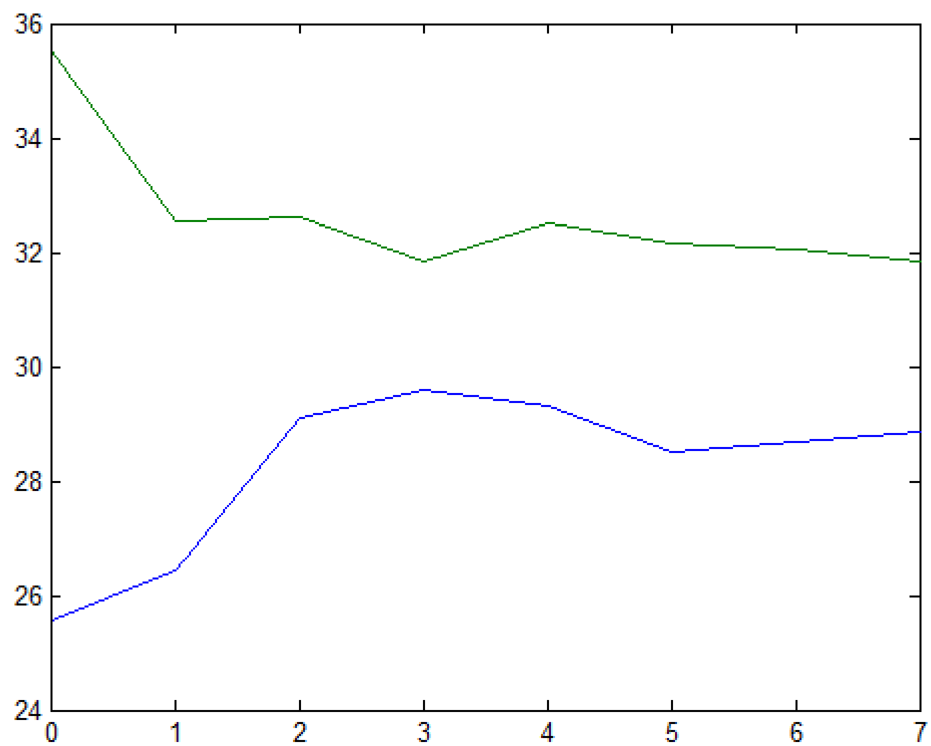
4. Volatility Trading

4.1. VIX Term Structure

In order to further examine the merits of volatility as its own asset class, we now turn to products available to the average investor. We have already looked at VX futures, which we used for our LV and VRP strategies in the portfolio optimization. However, the VIX calculation explained in the introduction demonstrates how convoluted the intuition behind VIX futures can be. For example, a three month VIX future is more directly measuring what S&P 500 options traders expect for market conditions to be up to five months from now.

As with any futures, traders will be most interested in the roll return and spot return of this product. The roll return depends crucially on whether the term structure of VIX demonstrates contango or backwardation. In Figure 5, we show the VIX term structure of two different dates. On August 3, 2009, the term structure is in contango (blue); it is upward sloping (particularly sensitive in the short term) and the futures prices are higher than the spot price. On the other hand, the term structure on June 1, 2010 is in backwardation (green). Again the short term is much more sensitive, but now the futures prices are lower than the VIX spot price, hence a downward slope. The VIX spot price has a high volatility and as such it moves around a great deal. These movements and spikes do not usually affect the futures price as much as changes in the trend of VIX (high volatility versus low volatility regimes).

Figure 5: VIX Term Structure on August 3, 2009 shown in blue and on June 1, 2010 shown in green



The term structure is generally determined by supply and demand factors of the market. But since VIX is not a tradable asset, we have to look at how the volatility is actually being traded. The implied volatility for the VIX calculations comes from S&P 500 options. The futures of different maturities will reflect the implied volatility of options at their respective maturity. As an option nears its expiration date, its theta or time sensitivity increases, and hence its value falls continually. A bullish market will have relatively low volatility due to the asymmetry of the leverage effect, which is beneficial to the option seller. For this reason, we can expect more shorting in short-term options than long-term options. Since option sellers are short volatility, this would explain the lower value of short-term volatility futures as seen in contango. Moreover, in a bullish market, traders would be willing to pay a premium for volatility futures as it is a hedge against possible declines in the market (and increases in volatility): if the market crashes, the VIX futures will have a high payoff. This is also in line with the fact that VIX is known to be mean reverting.

In contrast, a bearish market will have high volatility, especially as panic hits investors causing overreactions. Options investors will now proceed to purchase options close to the strike in the short term, as they expect volatility to rise. These options, in particular, are the cheapest and have the most sensitivity to their underlier. Since the VIX calculation is based on the implied volatility of those short-term options that are close to their strike, it is clear this behavior will push the VIX up. This will cause the futures in the short term to be at a premium leading to backwardation. In a high volatility regime, with a bearish market, investor’s selling VIX futures would expect the mean reverting nature of volatility to bring the VIX back down and expect to receive a positive payoff.

To quantify this rationale behind contango and backwardation, we begin by performing a linear regression of VIX spot price on future spreads using data from January 2, 1990 and July 15, 2011. If bullish and bearish markets have a direct effect on the term structure, the regression coefficients should reflect this. Figure 6 shows the regression line of VIX versus the realized spread between spot and 1 month VIX (calculated as 1 month minus the spot). The beta coefficient (-0.501), which can be found in Table 24, is significant and negative. Thus, a higher spot VIX (bearish market) leads to a lower, negative spread (backwardation) as we described above. In Figure 7 and Table 25, the same results can be drawn for a comparison between VIX versus the realized spread between 1 month and 2 month VIX, giving further evidence to the argument. Table 26 shows the average realized spread (between spot and 1 month VIX) for when VIX is above and below its mean. There is a positive premium when the spot VIX is below the mean (and a negative premium when spot VIX is above the mean), demonstrating mean reversion. In Table 27, we divide the VIX into smaller bins. We can see that the there is a higher degree of backwardation when the VIX is higher and similar high levels of contango with lower levels of VIX. As such, a change in the term structure signifies a major change in market conditions.

Figure 6: Regression of VIX and realized spread between spot and 1 month VIX, January 2, 1990 – July 15, 2011

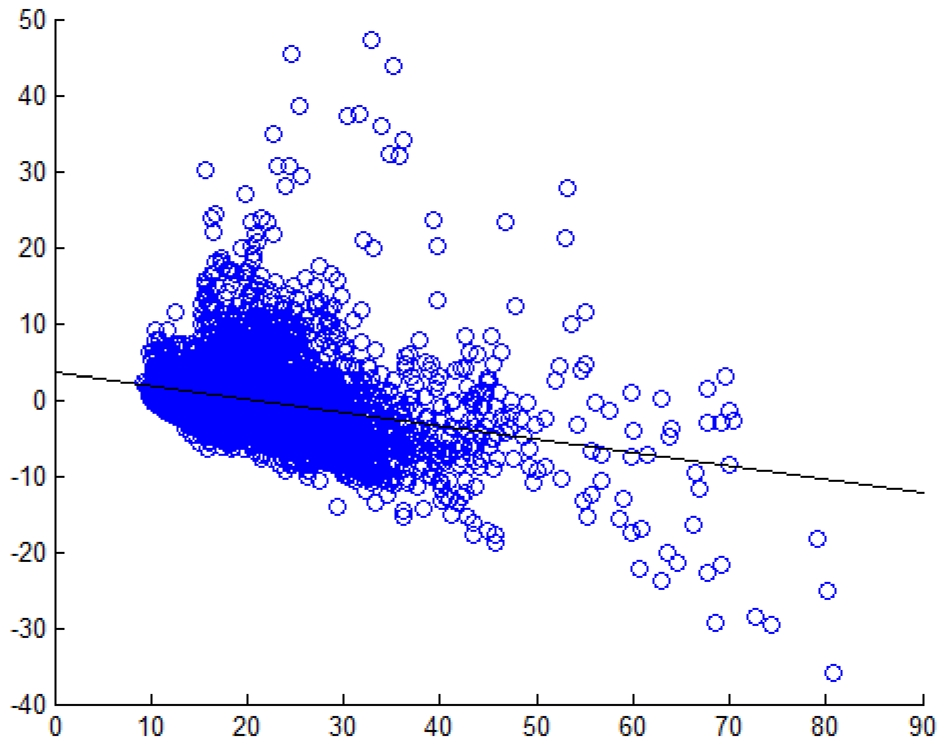


Table 24: Regression results of VIX and realized spread between spot and 1 month VIX, January 2, 1990 – July 15, 2011

	Alpha	Beta
Coefficient	20.341	-0.501
t stat	190.927	-22.811

Figure 7: Regression of VIX and realized spread between 1 month and 2 month VIX, January 2, 1990 – July 15, 2011

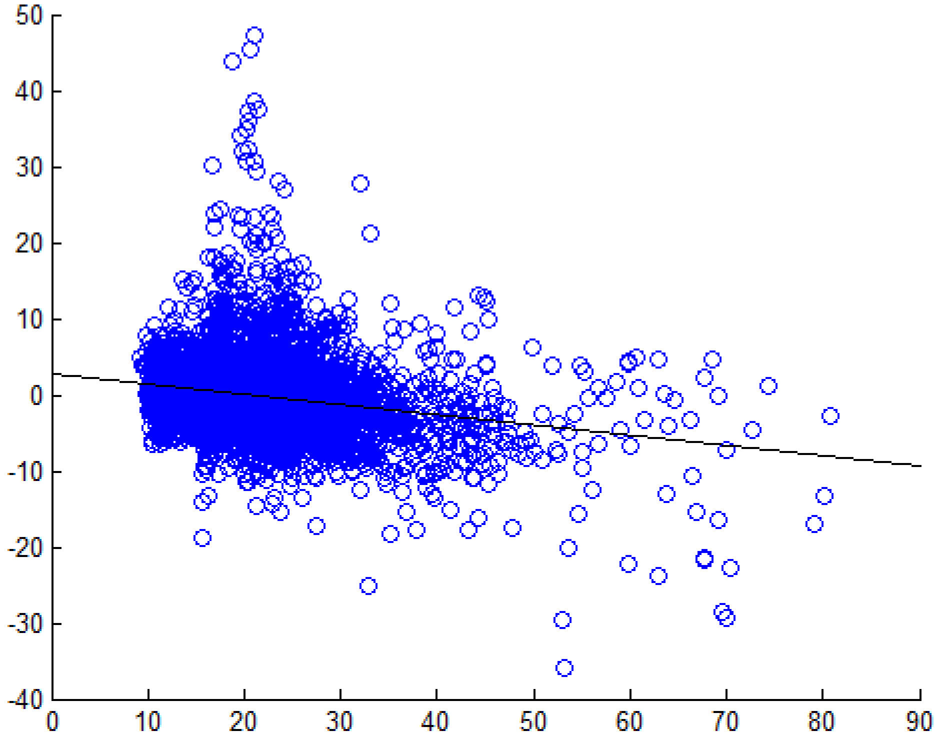


Table 25: Regression results of VIX and realized spread between 1 month and 2 month VIX, January 2, 1990 – July 15, 2011

	Alpha	Beta
Coefficient	20.347	-0.385
t stat	187.296	-17.176

Table 26: Average realized spread of VIX when above and below mean, January 2, 1990 – July 15, 2011

	Spot to 1 Month	1 Month to 2 Month
Less than Mean	0.720	0.436
Greater Than Mean	-1.036	-0.634

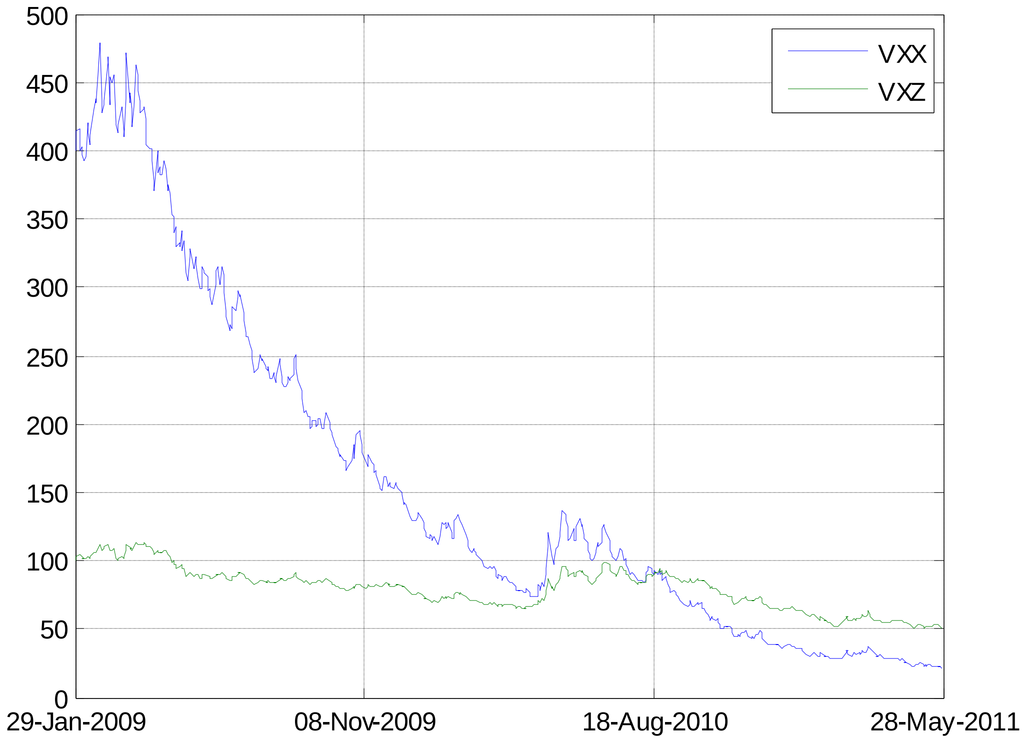
Table 27: Average realized spread of VIX at different levels, January 2, 1990 – July 15, 2011

	Spot to 1 Month	1 Month to 2 Month
Less than 15	0.568	0.293
15 to 20	0.821	0.556
20 to 25	0.594	0.745
25 to 30	-1.617	-0.871
30 to 35	-3.257	-2.433
35 to 40	-2.912	-3.643
40 to 45	-5.089	-3.820
Greater than 45	-6.172	-6.337

4.2. Volatility Futures

The advent of volatility future ETFs and ETNs have made strategies based on the above concepts accessible to average investors. Two of the more popular products are VXX, S&P 500 VIX Short-Term ETN, and VXZ, S&P 500 VIX Mid-Term ETN shown in Figure 8. Both products have been under much scrutiny mainly for their large losses since inception on January 29, 2009. These losses can be almost directly attributed to the declining volatility levels and the fact that the VIX term structure has been in constant contango causing negative roll returns in the recent period. Prior to inception, the term structure was in backwardation and would have affected the futures with positive roll return. Just as their VIX underlier, these ETNs are a hedge against extreme events that are not readily predicted. The largest visible spike in the price is just prior to the Greek protests around May 2010. Due to their limited history, it is difficult to estimate the degree of upside possible during such rare events but it is clear such products should follow market rules of no arbitrage and any apparent “free lunches” are not without subsequent risk.

Figure 8: Historical prices of VXX and VXZ, January 29, 2009 – May 27, 2011



4.3. Volatility Conditions

It is possible to surmise that the recent financial crisis has had a significant impact on the data we use, giving more extreme values than expected. Furthermore, examining volatility is most crucial during times of market downturns. Figure 9 highlights the key spikes in the VIX and relates them to the "black swan" events that caused such extreme movements. From these peaks, we can estimate the time it takes for the market to return to "normalcy", by finding when the VIX reverts back to its mean levels. In Table 28, the number of days it takes for the VIX to return to its mean value after a crisis is shown. The dates have been selected by when there is a peak in the VIX. The average time it takes for a large scale crisis to quell is about 138 business days or approximately 6 months. It is important to note we are simply measuring the speed of mean reversion and this does not necessarily reflect an economic recovery. Naturally, the magnitude of the crisis has a clear impact as can be seen by the lengthy recovery of the recent financial crisis. The LTCM bailout and the Asian Crisis had lesser impacts on volatility probably due to the fact that they were incidents that did not affect the entire US economy directly. Table 29 shows a similar chart with the average number of days it took for the VIX to reach mean levels after hitting a certain percentile. After reaching the 10th percentile, it took approximately 349 business days for VIX to return to the mean and when at the 90th percentile, the VIX took about 94 business days to bounce back. This demonstrates that high volatility appears in spikes but tends to subside quicker than low volatility, where the markets stay calmer for longer periods.

Figure 9: VIX and key events leading to large spikes, January 2, 1990 – July 15, 2011

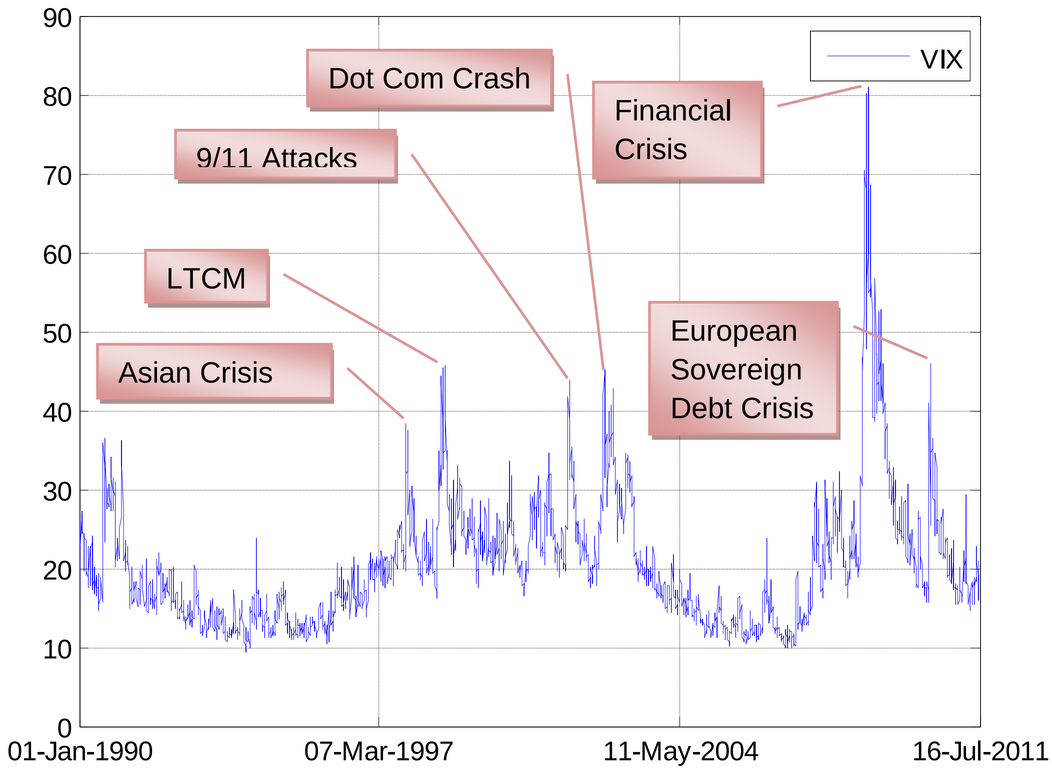


Table 28: Speed of mean reversion of VIX for specific events, January 2, 1990 – July 15, 2011

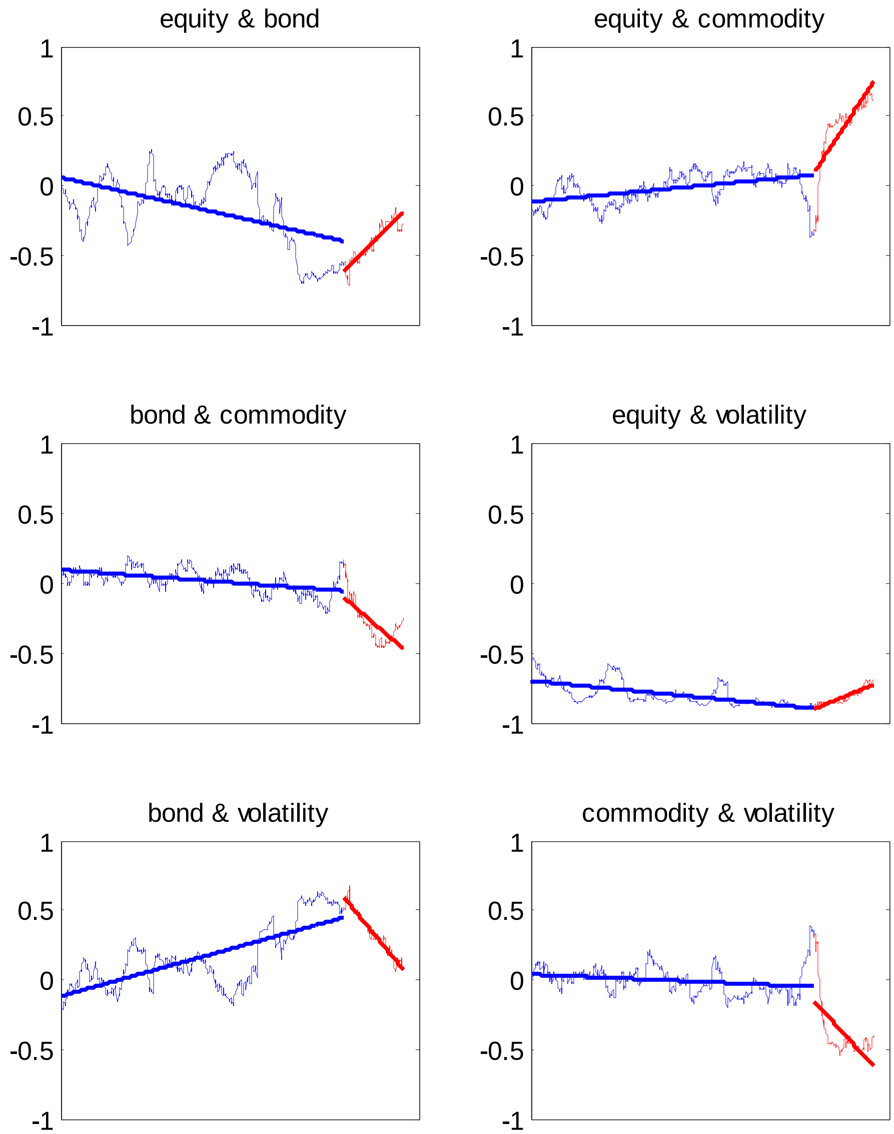
	Time in Days	Date	VIX
Asian Crisis	70	October 30, 1997	38.2
LTCM	53	October 8, 1998	45.74
September 11 Attacks	114	September 17, 2001	41.76
Dot Com Crash	190	July 23, 2002	44.92
Financial Crisis	292	October 24, 2008	79.13
European Sovereign Debt Crisis	108	May 7, 2010	40.95
Average	138		

Table 29: Speed of mean reversion of VIX for percentiles, January 2, 1990 – July 15, 2011

	Average Time in Days
5%	331
10%	349
25%	187
75%	61
90%	94
95%	116

When examining volatility conditions, it is also important to examine the correlations in the market. Generally, volatility and correlation both increase in tandem. When market conditions deteriorate, the volatility increases and all assets have extreme movements in tandem, hence increasing correlation. Likewise, as markets return to normal, the correlations will decrease as will volatility. Figure 10 shows the rolling 50-day correlations among the various asset classes: equities, bonds, commodities, and volatility, from April 1, 2003 (shortly after the Dot Com Bubble) to September 22, 2009. We have split the data into two sections with the first period (blue) leading up to the financial crisis until October 24, 2008 (determined using the peak of the VIX from above) and the second period (red) containing the crisis and subsequent recession. The equities & bonds correlation is generally negative and peaks to its lowest value at the onset of the crisis. During the high volatility regime, the correlation goes back from its extreme toward zero. In contrast, the next two graphs of equities & commodities and bonds & commodities, both have just about zero correlation during low volatility periods. However at the onset of the crisis, the correlation starts trending towards the extremes and seems to remain there at the end of the sample period. This is interesting because even though volatility decreases after its peak on October 24, 2008, the correlations continue climbing steadily to their extremes which is a behavior not usually seen. The implications for a portfolio containing those assets arise from the fact that low correlations near zero moderate the volatility of a portfolio. If an investor does not anticipate this structural change to extreme correlations, the volatility of the portfolio will increase even further in a high volatility regime.

Figure 10: Rolling 50-day correlations between asset classes divided on October 24, 2008 with pre-crisis shown in blue and post-crisis shown in red, April 1, 2003 – September 22, 2009



The next three graphs show the correlation of LV with each of the assets. This is important to examine to see if the crisis has caused any structural changes which could affect our portfolio optimization. The correlation with equity remains extremely negative which is a sign that LV is an excellent hedge for equities regardless of market condition. This is also expected as the LV strategy is based on the VIX which itself has the S&P 500 as an underlier so the dependence is very direct in this case. The correlation between bonds and volatility peaks at the crisis but then reverts to zero as before. For volatility and commodities, there also seems to be a structural change in the high volatility regime that leads to the two becoming negatively correlated.

5. Conclusion

Recent innovations in volatility products have made volatility trading accessible to most investors. These investments in volatility are gaining rapid popularity due to their behaviors and benefits in portfolio optimization. Traditional portfolios will generally consist largely of equities and bonds along with smaller holdings in other asset classes. When examining the performance of these portfolios, investors will concentrate on risk-adjusted returns as well as correlations among asset classes and changes in higher-order moments of the portfolio. We demonstrated that the inclusion of volatility as an asset class, via long volatility and long volatility risk premium, can add both diversification benefits and return enhancements, respectively. The initial analysis shows that each volatility strategy alone extends the efficient frontier out, but due to their negative correlation with each other, a combination of the two strategies has a significant improvement on the efficient frontier.

Dividing the dataset into a low volatility and high volatility regime demonstrates that in either period the addition of volatility to the asset allocation is greatly beneficial as can be measured by risk-adjusted returns. Using econometric methods to extend the dataset give similar results on the advantages of including volatility. Unfortunately, the data is still limited to only about 20 years and we cannot expect it to capture all types of “black swan” events which reflect the true nature of volatility.

Since there are many new volatility products with a limited history for investors to choose from, we showed the importance of understanding the underlying factors. For volatility futures, the VIX term structure is crucial due to recent contango that has greatly reduced value of these products. We have shown that changes in the term structure are generally a sign of events that have significantly changed market conditions. Finally, this analysis has highlighted that high volatility regimes have effects on the correlations among asset classes, which also affects portfolio performance. Further research can examine the addition of correlation strategies which is often tied closely with volatility. Overall, the benefits of volatility as an asset class are made apparent and suggestions are made on factors investors should consider when examining volatility products.

6. References

Andersen, T. G., Bondarenko, O., & Gonzalez-Perez, M. T. (2011). *A Corridor Fix for VIX: Developing a Coherent Model-Free Option-Implied Volatility Measure.*

Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *The Journal of Political Economy , 81* (3), 637-654.

Carr, P., & Wu, L. (2009). Variance Risk Premiums. *Review of Financial Studies , 22* (3), 1311-1341.

Chicago Board Options Exchange. (2009). *The CBOE Volatility Index - VIX.*

Christie, A. A. (1982). The stochastic behavior of common stock variances : Value, leverage and interest rate effects. *Journal of Financial Economics , 10* (4), 407-432.

Chunhachinda, P., Dandapani, K., Hamid, S., & Prakash, A. J. (1997). Portfolio selection and skewness: Evidence from international stock markets. *Journal of Banking & Finance , 21* (2), 143-167.

Daigler, R. T., & Rossi, L. (2006). A Portfolio of Stocks and Volatility. *The Journal of Investing , 15* (2), 99-106.

Demeterfi, K., Derman, E., Kamal, M., & Zou, J. (1999). A Guide to Volatility and Variance Swaps. *The Journal of Derivatives , 6* (4), 9-32.

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy? *Review of Financial Studies , 22* (5), 1915-1953.

Driessen, J., Maenhout, P. J., & Vilkov, G. (2009). The Price of Correlation Risk: Evidence from Equity Options. *The Journal of Finance , 64* (3), 1377-1406.

Engle, R. F., & Ng, V. K. (1993). Measuring and Testing the Impact of News on Volatility. *Journal of Finance , 48* (5), 1749-1778.

Engle, R. F., & Patton, A. J. (2001). What good is a volatility model? *Quantitative Finance , 1* (2), 237-245.

Eraker, B. (2008). *The Volatility Premium.*

Federal Reserve Bank of St. Louis. (n.d.). *The Financial Crisis: A Timeline of Events and Policy Actions*. Retrieved August 2011, from The Financial Crisis Timeline: http://timeline.stlouisfed.org/index.cfm?p=timeline#

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the Relation between the Expected Value and the Volatility of the Nominal Excess Return on Stocks. *The Journal of Finance , 48* (5), 1779-1801.

Gorton, G., & Rouwenhorst, K. G. (2006). Facts and Fantasies about Commodity Futures. *Financial Analysts Journal , 62* (2), 47-68.

Hafner, R., & Wallmeier, M. (2007). Volatility as an Asset Class: European Evidence. *European Journal of Finance , 13* (7), 621-644.

Haugen, R. A., Talmor, E., & Torous, W. N. (1991). The Effect of Volatility Changes on the Level of Stock Prices and Subsequent Expected Returns. *The Journal of Finance , 46* (3), 985-1007.

Ivashina, V., & Scharfstein, D. (2010). Bank lending during the financial crisis of 2008. *Journal of Financial Economics , 97* (3), 319-338.

Jagannathan, R., & Ma, T. (2003). Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps. *The Journal of Finance , 58* (4), 1651-1684.

Jarque, C. M., & Bera, A. K. (1987). A Test for Normality of Observations and Regression Residuals. *International Statistical Review , 55* (2), 163-172.

Jorion, P. (1985). International Portfolio Diversification with Estimation Risk. *The Journal of Business , 58* (3), 259-278.

Kim, C.-J., Morley, J. C., & Nelson, C. R. (2004). Is There a Positive Relationship between Stock Market Volatility and the Equity Premium? *Journal of Money, Credit and Banking , 36* (3), 339-360.

King, A. J. (1993). Asymmetric risk measures and tracking models for portfolio optimization under uncertainty. *Annals of Operations Research , 45* (1), 165-177.

Lai, T.-Y. (1991). Portfolio selection with skewness: A multiple-objective approach. *Review of Quantitative Finance and Accounting , 1* (3), 293-305.

Ledoit, O., & Wolf, M. (2003). *Honey, I Shrunk the Sample Covariance Matrix.*

Mandelbrot, B. (1963). The Variation of Certain Speculative Prices. *The Journal of Business , 36* (4), 394-419.

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance , 7* (1), 77-91.

Schwert, W. G. (1989). Why Does Stock Market Volatility Change Over Time? *Journal of Finance , 44* (5), 1115-1153.

Sharpe, W. F. (1966). Mutual Fund Performance. *The Journal of Business , 39* (1), 119-138.

Signori, O., Briere, M., & Burgues, A. (2009). *Volatility as an Asset Class for Long-Term Investors.*

Sortino, F. A., & Price, L. N. (1994). Performance Measurement in a Downside Risk Framework. *The Journal of Investing , 3* (3), 59-64.

Tang, K., & Xiong, W. (2010). *Index Investment and Financialization of Commodities.*

Taylor, J. B. (2008). *The Financial Crisis and the Policy Responses: An Empirical Analysis of What Went Wrong.*

Treynor, J. L. (1965). How to Rate Management of Investment Funds. *Harvard Business Review , 43*, 63-75.

Wu, G. (2001). The Determinants of Asymmetric Volatility. *The Review of Financial Studies , 14* (3), 837-859.

7. Appendix A

7.1. Asset Analysis & Mean-Variance Optimization

```
%% Asset Analysis/Mean-Variance Optimization
% S&P500, GSCI, CBOEVPD, CVX, IEF (bonds)
gsci_d_ts=fints(datenum(GSCI{1}), GSCI{2}, 'GSCI');
sp500_d_ts=fints(datenum(SP500_d{1},'yyyymmdd'), SP500_d{2}(:,1), 'SP500');
vpd_d_ts=fints(datenum(VPD{1}), VPD{2}, 'VPD');
cvx_d_ts=fints(datenum(CVX{1}), CVX{2}, 'VX');
ief_d_ts=fints(datenum(IEF{1}), IEF{2}(:,6), 'IEF');
rf_d_ts=fints(datenum(RF{1}, 'yyyymmdd'), RF{2}, 'RF');

% datestr(ftsbound(gsci_d_ts))

% Figure 1
temp=mean(sp500_d_ts);
means(1)=temp.SP500;
temp=mean(ief_d_ts);
means(2)=temp.IEF;
temp=mean(gsci_d_ts);
means(3)=temp.GSCI;
temp=mean(vpd_d_ts);
means(4)=temp.VPD;
temp=mean(cvx_d_ts);
means(5)=temp.VX;

temp=std(sp500_d_ts);
stddevs(1)=temp.SP500;
temp=std(ief_d_ts);
stddevs(2)=temp.IEF;
temp=std(gsci_d_ts);
stddevs(3)=temp.GSCI;
temp=std(vpd_d_ts);
stddevs(4)=temp.VPD;
temp=std(cvx_d_ts);
stddevs(5)=temp.VX;

combined_fts=merge(...
    (sp500_d_ts-means(1))/stddevs(1),...
    (ief_d_ts-means(2))/stddevs(2),...
    (gsci_d_ts-means(3))/stddevs(3),...
    (vpd_d_ts-means(4))/stddevs(4),...
    (cvx_d_ts-means(5))/stddevs(5),...
    'DateSetMethod', 'intersection', 'SortColumns', 0);

plot(combined_fts)
% End Figure 1

combined_fts=merge(sp500_d_ts, ief_d_ts, gsci_d_ts, vpd_d_ts, cvx_d_ts,
'DateSetMethod', 'intersection', 'SortColumns', 0);
combined_ret=tick2ret(fts2mat(combined_fts));
assets1=combined_ret(:,1:3); %EBC
```

```
assets2=combined_ret(:,1:4); %EBC + VRP
assets3=[combined_ret(:,1:3) combined_ret(:,5)]; %EBC + LV
assets4=combined_ret; %EBC + VRP + LV

temp=ftsbound(combined_fts);
rf_d_ts=fetch(rf_d_ts, datestr(temp(1)),[],datestr(temp(2)),[],1,'d');
rf=(geomean(fts2mat(rf_d_ts)+1)-1);
annual_rf=(1+rf)^252-1;

% The default lower bound is all zeros (no short-selling), the default
% upper bound is all ones (any asset may comprise the entire portfolio)
[risk(:,1), ret(:,1), weights1]=frontcon(mean(assets1), cov(assets1),
100,[],[-1 -1 -1; 1 1 1]);
[risk(:,2), ret(:,2), weights2]=frontcon(mean(assets2), cov(assets2),
100,[],[-1 -1 -1 -1; 1 1 1 1]);
[risk(:,3), ret(:,3), weights3]=frontcon(mean(assets3), cov(assets3),
100,[],[-1 -1 -1 -1; 1 1 1 1]);
[risk(:,4), ret(:,4), weights4]=frontcon(mean(assets4), cov(assets4),
100,[],[-1 -1 -1 -1 -1; 1 1 1 1 1]);

% Table 5
% RiskyRisk
% RiskyReturn
% RiskyFraction
% OverallRisk
% OverallReturn
[table5(1,1), table5(2,1), riskyweight1, table5(3,1), table5(4,1)...
    , table5(5,1)]=portalloc(risk(:,1), ret(:,1), weights1, rf, rf);
[table5(1,2), table5(2,2), riskyweight2, table5(3,2), table5(4,2)...
    , table5(5,2)]=portalloc(risk(:,2), ret(:,2), weights2, rf, rf);
[table5(1,3), table5(2,3), riskyweight3, table5(3,3), table5(4,3)...
    , table5(5,3)]=portalloc(risk(:,3), ret(:,3), weights3, rf, rf);
[table5(1,4), table5(2,4), riskyweight4, table5(3,4), table5(4,4)...
    , table5(5,4)]=portalloc(risk(:,4), ret(:,4), weights4, rf, rf);

% Final wealth starting with 100
sum(riskyweight4*100.*prod(1+combined_ret));

% Figure 4
plot(risk(:,1), ret(:,1),'b')
hold on
plot(risk(:,2), ret(:,2),'c') %VRP
plot(risk(:,3), ret(:,3),'r') %LV
plot(risk(:,4), ret(:,4),'g')

% End Figure 4

% Table 1
% Col: Equity, Bond, Commodity, VRP, LV
%Daily geom. mean
table1(1,:)=(geomean(assets4+1)-1)*100;
% annual geo mean
table1(2,:)=(geomean(assets4+1).^252-1)*100;
% max daily gain
table1(3,:)=max(assets4)*100;
```

```
% max daily loss
table1(4,:)=min(assets4)*100;
% ann. std dev
table1(5,:)=std(assets4)*sqrt(252)*100;
% skewness
table1(6,:)=skewness(assets4);
% kurtosis
table1(7,:)=kurtosis(assets4);
% success rate
table1(8,:)=sum(assets4>0)/size(assets4,1)*100;
% Sharpe Ratio
table1(9,:)=(table1(2,:)-annual_rf*100)./table1(5,:);
% Normality test
for i=1:5
    table1(12,i)=jbtest(assets4(:,i));
end

% Treynor Ratio
[tp1, tp2, tp3]=frontcon(mean(assets4), cov(assets4),100,[],[0 0 0 0 0; 1 1 1
1 1]);
[tp4, tp5, riskyweight5, tp6, tp7, tp8]=portalloc(tp1, tp2, tp3, rf, rf);
clearvars tp1 tp2 tp3 tp4 tp5 tp6 tp7 tp8;
tp=(riskyweight5*assets4')';
for i=1:5
    table1(10,i)=regress(assets4(:,i)-rf, tp-rf);
    table1(11,i)=(table1(2,i)-annual_rf*100)./regress(assets4(:,i)-rf, tp-
rf);
end

% End Table 1

% Table 2
table2=corr(assets4);

% More Table 5
tp=(riskyweight1*assets1')';
table6(1,1)=(geomean(tp+1)-1)*100;
table6(2,1)=(geomean(tp+1).^252-1)*100;
table6(3,1)=max(tp)*100;
table6(4,1)=min(tp)*100;
table6(5,1)=std(tp)*sqrt(252)*100;
table6(6,1)=skewness(tp);
table6(7,1)=kurtosis(tp);
table6(8,1)=sum(tp>0)/size(tp,1)*100;

tp=(riskyweight2*assets2')';
table6(1,2)=(geomean(tp+1)-1)*100;
table6(2,2)=(geomean(tp+1).^252-1)*100;
table6(3,2)=max(tp)*100;
table6(4,2)=min(tp)*100;
table6(5,2)=std(tp)*sqrt(252)*100;
table6(6,2)=skewness(tp);
table6(7,2)=kurtosis(tp);
table6(8,2)=sum(tp>0)/size(tp,1)*100;
```

```
tp=(riskyweight3*assets3')';
table6(1,3)=(geomean(tp+1)-1)*100;
table6(2,3)=(geomean(tp+1).^252-1)*100;
table6(3,3)=max(tp)*100;
table6(4,3)=min(tp)*100;
table6(5,3)=std(tp)*sqrt(252)*100;
table6(6,3)=skewness(tp);
table6(7,3)=kurtosis(tp);
table6(8,3)=sum(tp>0)/size(tp,1)*100;

tp=(riskyweight4*assets4')';
table6(1,4)=(geomean(tp+1)-1)*100;
table6(2,4)=(geomean(tp+1).^252-1)*100;
table6(3,4)=max(tp)*100;
table6(4,4)=min(tp)*100;
table6(5,4)=std(tp)*sqrt(252)*100;
table6(6,4)=skewness(tp);
table6(7,4)=kurtosis(tp);
table6(8,4)=sum(tp>0)/size(tp,1)*100;

table6(9,:)=(table6(2,:)-annual_rf*100)./table6(5,:);

%NP estimation - Figure 3
ksdensity(tp);
hold on
ix = -6*std(tp):1e-3:6*std(tp); %covers more than 99% of the curve
iy = pdf('normal', ix, mean(tp), std(tp));
plot(ix,iy,'r');
%

% Wealth graph
t1=cumprod(assets4(:,1)+1)*100*riskyweight4(1);
t2=cumprod(assets4(:,2)+1)*100*riskyweight4(2);
t3=cumprod(assets4(:,3)+1)*100*riskyweight4(3);
t4=cumprod(assets4(:,4)+1)*100*riskyweight4(4);
t5=cumprod(assets4(:,5)+1)*100*riskyweight4(5);
tot=t1+t2+t3+t4+t5;
plot([t1 t2 t3 t4 t5])
area([t1./tot t2./tot t3./tot t4./tot t5./tot])
```

7.2. Mean-Variance Optimization – High/Low Volatility Regimes

```
%% Efficient frontier S&P500, GSCI, CBOEVPD, CVX, IEF (bonds) low/high vol
gsci_d_ts=fints(datenum(GSCI{1}), GSCI{2}, 'GSCI');
sp500_d_ts=fints(datenum(SP500_d{1},'yyyymmdd'), SP500_d{2}(:,1), 'SP500');
vpd_d_ts=fints(datenum(VPD{1}), VPD{2}, 'VPD');
cvx_d_ts=fints(datenum(CVX{1}), CVX{2}, 'CVX');
ief_d_ts=fints(datenum(IEF{1}), IEF{2}(:,6), 'IEF');
rf_d_ts=fints(datenum(RF{1}, 'yyyymmdd'), RF{2}, 'RF');
vix_d_ts=fints(datenum(VIX2{1}), VIX2{2}(:,4),'VIX');
```

```
cutoff=20;

% Volatility Regimes
VIX2{2}(:,4);
% Dates of full analysis (based on all time series)
% 15-Jun-2004
% 31-Dec-2010
vix_d_ts=vix_d_ts(3642:5292);
%
plot(tsmovavg(vix_d_ts,'s',50));
tp=fts2mat(tsmovavg(vix_d_ts,'s',50));
tp2=tp>cutoff;

for i=1:size(tp2,1)-1
    if tp2(i)<tp2(i+1)
        data1(i)=50;
    else
        data1(i)=0;
    end
end
data1(size(tp2,1))=0;

for j=1:size(tp2,1)-1
    if tp2(j)>tp2(j+1)
        data2(j)=50;
    else
        data2(j)=0;
    end
end
data2(size(tp2,1))=0;


tb1=find(data1==50); % low to high
tb2=find(data2==50); % high to low
regime=zeros(size(tp,1),2);
for i=1:size(tb1,2)
    regime(tb1(i):tb2(i)-1,1)=1;
end

for i=1:size(tb2,2)-1
    regime(tb2(i):tb1(i+1)-1,2)=1;
end

regime(tb2(end):end,2)=1; % started w/ low vol
regime(1:tb1(1)-1,2)=1; % ended w/ low vol

% column 1 high vol
% column 2 low vol

% End Vol regimes

% Actual dates
for i=1:size(tb1,2)
    tp=fts2mat(vix_d_ts(tb1(i)),1);
    datestr(tp(1))
```

```
end
for i=1:size(tb2,2)
    tp=fts2mat(vix_d_ts(tb2(i)),1);
    datestr(tp(1))
end

cutoff_date=733281; %from above

combined_fts=merge(sp500_d_ts, ief_d_ts, gsci_d_ts, vpd_d_ts, cvx_d_ts,
'DateSetMethod', 'intersection', 'SortColumns', 0);
combined_ret=tick2ret(fts2mat(combined_fts));
assets_all=combined_ret; %BEC + VRP + LV
assets1=combined_ret(1:tb1(1)-1,:); %low
assets2=combined_ret(tb1(1)-1:end,:); %high

temp=ftsbound(combined_fts);
rf_d_ts_tp=fetch(rf_d_ts, datestr(temp(1)),[],datestr(temp(2)),[],1,'d');
rf=(geomean(fts2mat(rf_d_ts_tp)+1)-1);

[risk(:,1), ret(:,1), weights1]=frontcon(mean(assets1), cov(assets1),
100,[],[-1 -1 -1 -1 -1; 1 1 1 1 1]);
[risk(:,2), ret(:,2), weights2]=frontcon(mean(assets2), cov(assets2),
100,[],[-1 -1 -1 -1 -1; 1 1 1 1 1]);
[risk(:,3), ret(:,3), weights3]=frontcon(mean(assets_all), cov(assets_all),
100,[],[-1 -1 -1 -1 -1; 1 1 1 1 1]);

% RiskyRisk
% RiskyReturn
% RiskyFraction
% OverallRisk
% OverallReturn
[table5(1,1), table5(2,1), riskyweight1, table5(3,1), table5(4,1)...
    , table5(5,1)]=portalloc(risk(:,1), ret(:,1), weights1, rf, rf);
[table5(1,2), table5(2,2), riskyweight2, table5(3,2), table5(4,2)...
    , table5(5,2)]=portalloc(risk(:,2), ret(:,2), weights2, rf, rf);
[table5(1,3), table5(2,3), riskyweight3, table5(3,3), table5(4,3)...
    , table5(5,3)]=portalloc(risk(:,3), ret(:,3), weights3, rf, rf);

% final wealth
wealth=sum(riskyweight1*100.*prod(1+combined_ret(1:tb1(1)-1,:)))
wealth=sum(riskyweight2*wealth.*prod(1+combined_ret(tb1(1)-1:end,:)))

temp=ftsbound(combined_fts);
tp=fts2mat(vix_d_ts(tb1(i)),1);
rf_d_ts_tp=fetch(rf_d_ts, datestr(temp(1)),[],datestr(cutoff_date),[],1,'d');
rf=(geomean(fts2mat(rf_d_ts_tp)+1)-1);
annual_rf(1)=(1+rf)^252-1;
rf_d_ts_tp=fetch(rf_d_ts, datestr(cutoff_date),[],datestr(temp(2)),[],1,'d');
rf=(geomean(fts2mat(rf_d_ts_tp)+1)-1);
annual_rf(2)=(1+rf)^252-1;
rf_d_ts_tp=fetch(rf_d_ts, datestr(temp(1)),[],datestr(temp(2)),[],1,'d');
rf=(geomean(fts2mat(rf_d_ts_tp)+1)-1);
annual_rf(3)=(1+rf)^252-1;

tp=(riskyweight1*assets1')';
```

```
table6(1,1)=(geomean(tp+1)-1)*100;
table6(2,1)=(geomean(tp+1).^252-1)*100;
table6(3,1)=max(tp)*100;
table6(4,1)=min(tp)*100;
table6(5,1)=std(tp)*sqrt(252)*100;
table6(6,1)=skewness(tp);
table6(7,1)=kurtosis(tp);
table6(8,1)=sum(tp>0)/size(tp,1)*100;
table6(9,1)=(table6(2,1)-annual_rf(1)*100)./table6(5,1);

tp=(riskyweight2*assets2')';
table6(1,2)=(geomean(tp+1)-1)*100;
table6(2,2)=(geomean(tp+1).^252-1)*100;
table6(3,2)=max(tp)*100;
table6(4,2)=min(tp)*100;
table6(5,2)=std(tp)*sqrt(252)*100;
table6(6,2)=skewness(tp);
table6(7,2)=kurtosis(tp);
table6(8,2)=sum(tp>0)/size(tp,1)*100;
table6(9,2)=(table6(2,2)-annual_rf(2)*100)./table6(5,2);

tp=(riskyweight3*assets_all')';
table6(1,3)=(geomean(tp+1)-1)*100;
table6(2,3)=(geomean(tp+1).^252-1)*100;
table6(3,3)=max(tp)*100;
table6(4,3)=min(tp)*100;
table6(5,3)=std(tp)*sqrt(252)*100;
table6(6,3)=skewness(tp);
table6(7,3)=kurtosis(tp);
table6(8,3)=sum(tp>0)/size(tp,1)*100;
table6(9,3)=(table6(2,3)-annual_rf(3)*100)./table6(5,3);

%Table 8 - low vs high vol asset stats
% Col: Equity, Bond, Commodity, VRP, LV
MAR=(1+annual_rf(1))^(1/252)-1;
table8a(1,:)=(geomean(assets1+1)-1)*100;
table8a(2,:)=(geomean(assets1+1).^252-1)*100;
table8a(3,:)=max(assets1)*100;
table8a(4,:)=min(assets1)*100;
table8a(5,:)=std(assets1)*sqrt(252)*100;
table8a(6,:)=skewness(assets1);
table8a(7,:)=kurtosis(assets1);
table8a(8,:)=sum(assets1>0)/size(assets1,1)*100;
table8a(9,:)=(table8a(2,:)-annual_rf(1)*100)./table8a(5,:);
table8a(10,:)=(mean(assets1)-MAR)./sqrt(lpm(assets1,MAR,2));

% Treynor Ratio
rf=MAR;
[tp1, tp2, tp3]=frontcon(mean(assets1), cov(assets1),100,[],[0 0 0 0 0; 1 1 1
1 1]);
[tp4, tp5, riskyweight_tp, tp6, tp7, tp8]=portalloc(tp1, tp2, tp3, rf, rf);
clearvars tp1 tp2 tp3 tp4 tp5 tp6 tp7 tp8;
tp=(riskyweight_tp*assets1')';
for i=1:5
    table8a(11,i)=regress(assets1(:,i)-rf, tp-rf);
```

```
    table8a(12,i)=(table8a(2,i)-annual_rf(1)*100)./regress(assets1(:,i)-rf,
tp-rf);
end

MAR=(1+annual_rf(2))^(1/252)-1;
table8b(1,:)=(geomean(assets2+1)-1)*100;
table8b(2,:)=(geomean(assets2+1).^252-1)*100;
table8b(3,:)=max(assets2)*100;
table8b(4,:)=min(assets2)*100;
table8b(5,:)=std(assets2)*sqrt(252)*100;
table8b(6,:)=skewness(assets2);
table8b(7,:)=kurtosis(assets2);
table8b(8,:)=sum(assets2>0)/size(assets2,1)*100;
table8b(9,:)=(table8b(2,:)-annual_rf(2)*100)./table8b(5,:);
table8b(10,:)=(mean(assets2)-MAR)./sqrt(lpm(assets2,MAR,2));

% Treynor Ratio
rf=MAR;
[tp1, tp2, tp3]=frontcon(mean(assets2), cov(assets2),100,[],[0 0 0 0 0; 1 1 1
1 1]);
[tp4, tp5, riskyweight_tp, tp6, tp7, tp8]=portalloc(tp1, tp2, tp3, rf, rf);
clearvars tp1 tp2 tp3 tp4 tp5 tp6 tp7 tp8;
tp=(riskyweight_tp*assets2')';
for i=1:5
    table8b(11,i)=regress(assets2(:,i)-rf, tp-rf);
    table8b(12,i)=(table8b(2,i)-annual_rf(2)*100)./regress(assets2(:,i)-rf,
tp-rf);
end

% End table 8
```

7.3. Mean-Variance Optimization – Rebalancing

```
%% Efficient frontier S&P500, GSCI, CBOEVPD, CVX, IEF (bonds) w/ rebalancing
gsci_d_ts=fints(datenum(GSCI{1}), GSCI{2}, 'GSCI');
sp500_d_ts=fints(datenum(SP500_d{1},'yyyymmdd'), SP500_d{2}(:,1), 'SP500');
vpd_d_ts=fints(datenum(VPD{1}), VPD{2}, 'VPD');
cvx_d_ts=fints(datenum(CVX{1}), CVX{2}, 'CVX');
ief_d_ts=fints(datenum(IEF{1}), IEF{2}(:,6), 'IEF');
rf_d_ts=fints(datenum(RF{1}, 'yyyymmdd'), RF{2}, 'RF');
vix_d_ts=fints(datenum(VIX2{1}), VIX2{2}(:,4),'VIX');

combined_fts=merge(sp500_d_ts, ief_d_ts, gsci_d_ts, vpd_d_ts, cvx_d_ts,
'DateSetMethod', 'intersection', 'SortColumns', 0);
combined_ret=tick2ret(fts2mat(combined_fts));

rbf=30; % number of days before rebalancing, rebalance factor
combined_fts=combined_fts(2:end);
tc=.001; % .01% transaction costs for commission and slippage

% Dates
all_dates=fts2mat(combined_fts,1);
all_dates=all_dates(:,1);
```

```
n=7;
wealth=zeros(size(combined_ret,1)/rbf,7);
MAR=8.8531e-005;

% Change constraints below as necessary
table=zeros(5,size(combined_ret,1)/rbf);
riskyweights=zeros(size(combined_ret,1)/rbf, 5);
period_ret=zeros(size(combined_ret,1)/rbf, 5);


for i=1:size(combined_ret,1)/rbf
    dates(i,1) = all_dates((i-1)*rbf+1);
    assets=combined_ret((i-1)*rbf+1:i*rbf,:);
    period_ret(i,:)=prod(1+assets);
    [risk(:,i), ret(:,i), weights]=frontcon(mean(assets), cov(assets),
100,[],[0 0 0 0 0; 1 1 1 1 1]);

    temp=ftsbound(combined_fts((i-1)*rbf+1:i*rbf));
    tp=fetch(rf_d_ts, datestr(temp(1)),[],datestr(temp(2)),[],1,'d');
    if (geomean(fts2mat(tp)+1)-1)~=0
        rf=(geomean(fts2mat(tp)+1)-1);
    end

    [table(1,i), table(2,i), riskyweights(i,:), table(3,i), table(4,i)...
    , table(5,i)]=portalloc(risk(:,i), ret(:,i), weights, rf, rf);
    if i==1
        wealth(i,1)=100;
    else
        wealth(i,1)=sum(riskyweights(i-1,:)*wealth(i-
1,1).*period_ret(i,:))*(1-tc);
    end
end

clearvars tp;

% analysis
for i=1:size(combined_ret,1)/rbf
    tb((i-1)*rbf+1:i*rbf,:)=repmat(riskyweights(i,:),rbf,1);
end
tp=sum(tb(1:end-rbf,:).*combined_ret(rbf+1:end,:),2);

temp=ftsbound(combined_fts);
rf_d_ts=fetch(rf_d_ts, datestr(temp(1)),[],datestr(temp(2)),[],1,'d');
rf=(geomean(fts2mat(rf_d_ts)+1)-1);
annual_rf=(1+rf)^252-1;

table6(1,1)=(geomean(tp+1)-1)*100;
table6(2,1)=(geomean(tp+1).^252-1)*100;
table6(3,1)=max(tp)*100;
table6(4,1)=min(tp)*100;
table6(5,1)=std(tp)*sqrt(252)*100;
table6(6,1)=skewness(tp);
table6(7,1)=kurtosis(tp);
```

```
table6(8,1)=sum(tp>0)/size(tp,1)*100;
table6(9,1)=(table6(2,1)-annual_rf*100)./table6(5,1);
table6(10,1)=(mean(tp)-MAR)/sqrt(lpm(tp,MAR,2));
table6(11,1)=wealth(end,1);
```

7.4. Regime Varying Correlation Plots

```
%% Rolling window correlation
combined_fts=merge(sp500_d_ts, ief_d_ts, gsci_d_ts, vix_d_ts,
'DateSetMethod', 'intersection', 'SortColumns', 0);
combined_fts=fetch(combined_fts,
datestr(731672),[],datestr(734038),[],1,'d');
combined_ret=tick2ret(fts2mat(combined_fts));
%01-Apr-2003 to 22-Sep-2009
%after dot com
dates=ftsbound(combined_fts);
start=dates(1);
ending=dates(2);

% assuming financial crisis began oct 24, 2008
clf
cut=1367;

window=100;
cut=cut-window-1;
corr_mat=zeros(size(combined_ret,1)-window+1,6);
for i=1:size(combined_ret,1)-window+1
    tp=corr(combined_ret(i:i+window-1,:));
    corr_mat(i,:)=[tp(1,2) tp(1,3) tp(2,3) tp(1,4) tp(2,4) tp(3,4)];
end

names={'equity & bond';
    'equity & commodity';
    'bond & commodity';
    'equity & volatility';
    'bond & volatility';
    'commodity & volatility'};
for i=1:size(corr_mat,2)
    subplot(3,2,i)
    p1=plot(start+[1:cut],corr_mat(1:cut,i),'b');
    hold on
    p2=plot(start+[cut+1:size(corr_mat,1)],corr_mat(cut+1:end,i),'r');
    tb1=regress(corr_mat(cut+1:end,i),[ones(size(corr_mat,1)-cut,1)
[cut+1:size(corr_mat,1)]']);
    tb2=regress(corr_mat(1:cut,i),[ones(cut,1) [1:cut]']);

p3=plot(start+[cut+1:size(corr_mat,1)],tb1(1)+[cut+1:size(corr_mat,1)]'*tb1(2
));
    p4=plot(start+[1:cut],tb2(1)+[1:cut]'*tb2(2));
    set(gca,'XTick',[]);
    set(p3,'Color','red','LineWidth',2);
    set(p4,'Color','blue','LineWidth',2);
    title(names(i));
    axis([1+start 1600+start -1 1])
```

```
end

plot(fetch(vix_d_ts, datestr(731672),[],datestr(734038),[],1,'d'))
```

7.5. Extend Dataset via Regression

```
%% Extend VX
vix_d_ts=fints(datenum(VIX2{1}), VIX2{2}(:,4),'VIX');
cvx_d_ts=fints(datenum(CVX{1}), CVX{2}, 'VX');
combined=merge(vix_d_ts, cvx_d_ts, 'DateSetMethod', 'intersection',
'SortColumns', 0);

temp=fts2mat(combined);
tp=regstats(temp(:,2),temp(:,1));
cvx_d_ts=tp.beta(1)+tp.beta(2)*vix_d_ts;
cvx_d_ts=chfield(cvx_d_ts, 'VIX', 'VX');
% Jan 01, 1990 to July 15, 2011

[tp.beta(1) tp.beta(2) tp.adjrsquare]
[tp.tstat.t(1) tp.tstat.t(2)]
```

7.6. VIX Term Structure Analysis

```
%% VIX futures

vxx_d_ts=fints(datenum(VXX{1}), VXX{2}(:,2), 'VXX');
vxz_d_ts=fints(datenum(VXZ{1}), VXZ{2}(:,2), 'VXZ');

% Figure 5 - futures- contango & backwardation
tp=[7   28.85   31.85;
    6   28.7    32.05;
    5   28.5    32.15;
    4   29.3    32.51;
    3   29.6    31.85;
    2   29.1    32.6;
    1   26.45   32.55;
    0   25.56   35.54];
plot(tp(:,1), tp(:,2:3))

vix_d_ts=fints(datenum(VIX2{1}), VIX2{2}(:,4),'VIX');
vix=fts2mat(vix_d_ts);
month=22; % days

vix_mat=zeros(size(vix,1)-2*month, 3);
total=0;
for i=1:size(vix,1)-2*month
    vix_mat(i,:)=[vix(i) vix(i+month)-vix(i) vix(i+2*month)-vix(i+month)];
    if vix(i+month)-vix(i)<0
        total=total+1;
```

```
    end
end
total/(size(vix,1)-2*month)

bins=zeros(2,2);
count=zeros(2,1);
tp=mean(vix_d_ts);
for i=1:size(vix_mat,1)
    if vix_mat(i,1)<tp.VIX
        count(1)=count(1)+1;
        bins(1,1)=bins(1,1)+vix_mat(i,2);
        bins(1,2)=bins(1,2)+vix_mat(i,3);
    else
        count(2)=count(2)+1;
        bins(2,1)=bins(2,1)+vix_mat(i,2);
        bins(2,2)=bins(2,2)+vix_mat(i,3);
    end
end
bins=bins./[count count];

bins=zeros(8,2);
count=zeros(8,1);
for i=1:size(vix_mat,1)
    if vix_mat(i,1)<15
        count(1)=count(1)+1;
        bins(1,1)=bins(1,1)+vix_mat(i,2);
        bins(1,2)=bins(1,2)+vix_mat(i,3);
    elseif vix_mat(i,1)>=15 && vix_mat(i,1)<20
        count(2)=count(2)+1;
        bins(2,1)=bins(2,1)+vix_mat(i,2);
        bins(2,2)=bins(2,2)+vix_mat(i,3);
    elseif vix_mat(i,1)>=20 && vix_mat(i,1)<25
        count(3)=count(3)+1;
        bins(3,1)=bins(3,1)+vix_mat(i,2);
        bins(3,2)=bins(3,2)+vix_mat(i,3);
    elseif vix_mat(i,1)>=25 && vix_mat(i,1)<30
        count(4)=count(4)+1;
        bins(4,1)=bins(4,1)+vix_mat(i,2);
        bins(4,2)=bins(4,2)+vix_mat(i,3);
    elseif vix_mat(i,1)>=30 && vix_mat(i,1)<35
        count(5)=count(5)+1;
        bins(5,1)=bins(5,1)+vix_mat(i,2);
        bins(5,2)=bins(5,2)+vix_mat(i,3);
    elseif vix_mat(i,1)>=35 && vix_mat(i,1)<40
        count(6)=count(6)+1;
        bins(6,1)=bins(6,1)+vix_mat(i,2);
        bins(6,2)=bins(6,2)+vix_mat(i,3);
    elseif vix_mat(i,1)>=40 && vix_mat(i,1)<45
        count(7)=count(7)+1;
        bins(7,1)=bins(7,1)+vix_mat(i,2);
        bins(7,2)=bins(7,2)+vix_mat(i,3);
    else
        count(8)=count(8)+1;
        bins(8,1)=bins(8,1)+vix_mat(i,2);
        bins(8,2)=bins(8,2)+vix_mat(i,3);
    end
```

```
end
bins=bins./[count count];


%figure 6
subplot(1,2,1)
scatter(vix_mat(:,1), vix_mat(:,2))
lsline
%figure 7
subplot(1,2,2)
scatter(vix_mat(:,1), vix_mat(:,3))
lsline

tp=regstats(vix_mat(:,1), vix_mat(:,2));
tp.beta
tp.tstat.t

tp=regstats(vix_mat(:,1), vix_mat(:,3));
tp.beta
tp.tstat.t
```

7.7. VIX Speed of Mean Reversion Analysis

```
%% VIX speed of mean reversion - percentiles
vix
prctile(vix,10)
mean(vix)
prctile(vix,90)

count=0;
flag=0;
clearvars tp;
z=1;
for i=1:size(vix,1)
    if (vix(i)<prctile(vix,25) && (flag==0))
        count=i;
        tp(z,1)=count;
        flag=1;
    end
    if flag==1
        if vix(i)>mean(vix)
            flag=0;
            tp(z,2)=i-count;
            z=z+1;
        end
    end
end
mean(tp(:,2))

count=0;
flag=0;
clearvars tp;
z=1;
```

```
for i=1:size(vix,1)
    if (vix(i)>prctile(vix,95) && (flag==0))
        count=i;
        tp(z,1)=count;
        flag=1;
    end
    if flag==1
        if vix(i)<mean(vix)
            flag=0;
            tp(z,2)=i-count;
            z=z+1;
        end
    end
end
mean(tp(:,2))

%% VIX speed of mean reversion - events

vxx_d_ts=fints(datenum(VXX{1}), VXX{2}(:,2), 'VXX');
vxz_d_ts=fints(datenum(VXZ{1}), VXZ{2}(:,2), 'VXZ');
plot([vxx_d_ts vxz_d_ts])
% Figure 8

% Speed of mean reversion 2
fetch(vix_d_ts, '12-Jan-2011',[],'15-Feb-2011',[],1,'d')

vix=fts2mat(vix_d_ts, 1);

dates=['07-May-2010';
    '24-Oct-2008';
    '08-Oct-1998';
    '17-Sep-2001';
    '30-Oct-1997';
    '23-Jul-2002';];

mr=size(6,3);
for i=1:size(dates,1)
    count=1;
    start=find(vix==datenum(dates(i,:)));
    while vix(start+count,2) > mean(vix(:,2))
        count=count+1;
    end
    mr(i,1)=vix(start,1);
    mr(i,2)=count;
    mr(i,3)=vix(start,2);
end
```